A constrained joint link–matter Hamiltonian on the tetrahedral incidence graph

Gauss closure, reciprocal backreaction, global flow,
and an exact neutral curvature seed

Unified Dynamics research workstream | 15 October 2026 UTC
Version 0.1 | Conditional theorem manuscript

Abstract

We construct a finite classical Hamiltonian extension of the previously specified unitary Hasse-link transport family. A fundamental and a conjugate-representation matter field provide nonzero neutral initial data on a closed graph. The canonical moment map supplies first-class matrix Gauss constraints. The equations preserve those constraints, the Hamiltonian, and both matter norms; their temporal-gauge solutions exist globally. An explicit neutral scalar preparation starts with flat links and produces nonzero diamond curvature at second order in internal time. The scalar curvature defect begins as $2a^2s^4$. These are deductions within an explicitly chosen action, not a derivation of that action from UD, a selection of a physical gauge group, or evidence for force unification.

Claim typing. The Hamiltonian and extra matter species are CANDIDATE inputs. All universal statements below are DERIVED, CONDITIONAL ON THOSE INPUTS. Finite symbolic fixtures are EXACT constructor checks; an integration receipt is separately NUMERICAL. Physical promotion remains 0. This manuscript has not received independent review.

1 Source graph and transport convention

Let $K = \Delta^3$ be the filled tetrahedron, with its 15 nonempty simplices in increasing vertex orientation: 4 vertices, 6 edges, 4 faces and 1 tetrahedron. The signed total boundary B satisfies $B^2 = 0$. Its 28 nonzero entries define an oriented incidence graph. Write $e = (i \leftarrow j)$ when i is a codimension-one face of j, with $b_e = B_{ij} \in \{\pm 1\}$. Incidence signs are not electric-charge signs.

At each simplex place $\mathbb{C}^r$, $r \geq 1$, and at each oriented link place $U_e \in \mathrm{U}(r)$. Set

$$(D_U)_{ij} = b_e U_e, \qquad A_U = D_U^\dagger - D_U, \qquad h_U = iA_U = h_U^\dagger.$$

The old prescribed-link equation is $\dot{\psi} = A_U\psi$. Here links acquire their own momenta and equations. A prescribed identity connection is not automatically an invariant solution of the coupled model.

2 Declared canonical extension

At every link introduce a Hermitian left momentum E_e. At every node introduce independent complex classical fields $\psi_i, \chi_i \in \mathbb{C}^r$. They transform as

$$\psi_i' = g_i \psi_i, \qquad \chi_i' = \bar{g}_i \chi_i, \tag{1}$$

$$U_e' = g_i U_e g_j^\dagger, \qquad E_e' = g_i E_e g_i^\dagger, \qquad g_i \in \mathrm{U}(r). \tag{2}$$

The second field is an explicit extension. It is not yet identified with an antiparticle, a fermion, or a source-derived degree of freedom.

Use the product cotangent and canonical complex phase space, with real one-form

$$\Theta = -i \sum_e \operatorname{tr}(E_e \, \mathrm{d}U_e U_e^\dagger) + \frac{i}{2} \sum_i \left(\psi_i^\dagger \mathrm{d}\psi_i - \mathrm{d}\psi_i^\dagger \psi_i + \chi_i^\dagger \mathrm{d}\chi_i - \mathrm{d}\chi_i^\dagger \chi_i \right). \tag{3}$$

Take $\omega = -\mathrm{d}\Theta$, $\iota_{X_H}\omega = \mathrm{d}H$; thus $\{\psi_{i\alpha}, \bar{\psi}_{j\beta}\} = -i\delta_{ij}\delta_{\alpha\beta}$, and likewise for χ. Fields of different species commute.

A diamond $d = (i, j, h, k)$ has two paths from k to i:

$$P_d = U_{ij} U_{jk}, \qquad Q_d = U_{ih} U_{hk}, \qquad W_d = Q_d^\dagger P_d.$$

There are 18 diamonds. W_d is based at k and transforms by conjugation there. Choose constants $a > 0$, $\kappa \geq 0$, $\mu \geq 0$, and define

$$\begin{aligned} H = {} & \frac{a}{2} \sum_e \operatorname{tr}(E_e^2) + \kappa \sum_d \left(1 - \frac{1}{r} \operatorname{Re} \operatorname{tr} W_d \right) \\ & + \psi^\dagger (\mu \mathbf{1} + h_U) \psi + \chi^\dagger (\mu \mathbf{1} + \bar{h}_U) \chi. \end{aligned} \tag{4}$$

All quantities use dimensionless internal-time units at this stage. The parameters are action coefficients; no electromagnetic, weak, strong, gravitational, or laboratory normalization has been assigned. The on-site μ is a phase-rate coefficient, not an established rest mass.

Every term in (4) is real and gauge invariant. In particular $h_{U'} = G h_U G^\dagger$, where $G = \operatorname{diag}(g_i)$; complex conjugation gives the conjugate-representation covariance. The diamond term is nonnegative because W_d is unitary.

The canonical constrained action is

$$\mathcal{S} = \int \left[\Theta(\dot{x}) - H(x) - \sum_i \operatorname{tr}(\Lambda_i \mathcal{G}_i) \right] \mathrm{d}s, \qquad \Lambda_i = \Lambda_i^\dagger,$$

with $\mathcal{G}_i$ specified on the next page. This is a standard lattice Hamiltonian construction adapted to the incidence graph; the general cotangent mechanism is not claimed as new [1, 2].

3 Gauss constraints and the neutrality obstruction

Theorem 1 (Moment map and constrained closure)**.** *For the declared canonical phase space, the local gauge action has Hermitian moment map*

$$\mathcal{G}_i = \sum_{e:\,\mathrm{head}(e)=i} E_e - \sum_{e:\,\mathrm{tail}(e)=i} U_e^\dagger E_e U_e - \psi_i \psi_i^\dagger + \bar{\chi}_i \chi_i^T. \tag{5}$$

The constraints $\mathcal{G}_i = 0$ are first class and invariant under the Hamiltonian flow. Their zero set contains nonzero matter data. Its quotient may be stratified; no smooth free-action quotient is assumed.

Proof. For Hermitian node parameters T_i, the infinitesimal action is

$$\delta\psi_i = iT_i\psi_i, \quad \delta\chi_i = -i\bar{T}_i\chi_i, \quad \delta U_e = iT_iU_e - iU_eT_j.$$

Contracting Θ with this vector field gives exactly $\mathcal{J}[T] = \sum_i \mathrm{tr}(T_i\mathcal{G}_i)$: the link term contributes $\mathrm{tr}(T_iE_e) - \mathrm{tr}(T_jU_e^\dagger E_eU_e)$, and matter contributes $-\psi_i^\dagger T_i\psi_i + \chi_i^\dagger \bar{T}_i\chi_i$. The one-form is invariant, so Cartan's identity gives $\iota_{X_T}\omega = \mathrm{d}\mathcal{J}[T]$. Equivariance is direct: $\mathcal{G}_i' = g_i\mathcal{G}_ig_i^\dagger$. Thus

$$\{\mathcal{J}[T], \mathcal{J}[S]\} = \mathcal{J}[i[T,S]], \qquad \{\mathcal{J}[T], H\} = 0.$$

There is no central term. The first identity proves first-class closure; the second follows from the established invariance of H and proves preservation of the constraint surface. Finally, arbitrary ψ_i, $\chi_i = \bar{\psi}_i$, $E_e = 0$, and arbitrary unitary links give $\mathcal{G}_i = 0$ pointwise. This relation between the two species is an initial preparation, not an asserted invariant submanifold. □

Define $N_\psi = \sum_i \|\psi_i\|^2$ and $N_\chi = \sum_i \|\chi_i\|^2$. Taking traces and summing (5) cancels every link contribution:

$$\sum_i \mathrm{tr}\,\mathcal{G}_i = N_\chi - N_\psi. \tag{6}$$

Consequently zero-source Gauss law requires equal total norms. Equal totals alone do not solve every matrix constraint; the explicit preparation above does.

Proposition 2 (Scoped single-species obstruction)**.** *With this canonical fundamental matter field, full* $\mathrm{U}(r)$ *gauge group, no χ, no external charge, and the same closed graph, $\mathcal{G} = 0$ forces $\psi = 0$.*

Proof. Equation (6) becomes $0 = -N_\psi$; every summand in N_ψ is nonnegative. □

This does not exclude $\mathrm{SU}(r)$ theories, other matter phase spaces, boundaries, or explicit background sources. The repair chosen here adds a dynamical opposite-charge partner and keeps that assumption visible.

4 Equations and reciprocal backreaction

Work first in temporal gauge $\Lambda = 0$. Define the Hermitian left gradient $\mathcal{F}_e$ of all U-dependent terms by

$$\delta H|_{\delta U_e = iX_eU_e} = \sum_e \operatorname{tr}(\mathcal{F}_e X_e), \qquad X_e = X_e^\dagger.$$

The equations are

$$i\dot{\psi} = (\mu\mathbf{1} + h_U)\psi, \qquad i\dot{\chi} = (\mu\mathbf{1} + \bar{h}_U)\chi, \tag{7}$$

$$\dot{U}_e = iaE_eU_e, \qquad \dot{E}_e = -\mathcal{F}_e. \tag{8}$$

The electric coadjoint drift vanishes for the isotropic quadratic $\operatorname{tr} E_e^2$. Unitarity of U and Hermiticity of E are preserved. In particular $\dot{\psi} = A_U\psi - i\mu\psi$ while $\dot{\chi} = -\bar{A}_U\chi - i\mu\chi$.

Writing $z_i = \bar{\chi}_i$ only as a convenient force notation, the matter part is

$$\mathcal{F}_e^{\mathrm{m}} = b_e \left[U_e\psi_j\psi_i^\dagger + \psi_i\psi_j^\dagger U_e^\dagger + U_e z_j z_i^\dagger + z_i z_j^\dagger U_e^\dagger\right]. \tag{9}$$

Although z transforms as a fundamental vector, its conjugated symplectic structure has the opposite sign; treating it as a second ordinary canonical fundamental field would change the model. Equation (9) follows by differentiating the two real edge Hamiltonians, each of the form $2b_e \operatorname{Im}(\psi_i^\dagger U_e\psi_j)$ or $2b_e \operatorname{Im}(z_i^\dagger U_e z_j)$.

For an arbitrary temporal multiplier, add

$$\dot{\psi}_i \supset i\Lambda_i\psi_i, \quad \dot{\chi}_i \supset -i\bar{\Lambda}_i\chi_i, \quad \dot{U}_e \supset i\Lambda_iU_e - iU_e\Lambda_j, \quad \dot{E}_e \supset i[\Lambda_i, E_e].$$

Then $\dot{\mathcal{G}}_i = i[\Lambda_i, \mathcal{G}_i]$. Under a time-dependent gauge transformation, $\Lambda_i' = g_i\Lambda_i g_i^\dagger - i\dot{g}_i g_i^\dagger$. Thus the prior temporal connection is $\Omega_i = i\Lambda_i$ with its usual inhomogeneous transformation.

The scalar support currents for $e = (i \leftarrow j)$ are

$$J_e^\psi = 2b_e \operatorname{Re}(\psi_i^\dagger U_e\psi_j), \qquad J_e^\chi = 2b_e \operatorname{Re}(\chi_i^\dagger \bar{U}_e\chi_j).$$

That link contributes $(-J_e^\psi, +J_e^\psi)$ to the head/tail ψ stocks and $(+J_e^\chi, -J_e^\chi)$ to the χ stocks. Hence both norms are conserved. For central charge $q_i = \|\psi_i\|^2 - \|\chi_i\|^2$, the head loses $J_e^\psi + J_e^\chi$, exactly matching $\operatorname{tr}\dot{E}_e$ from the matter force. Matrix balance follows from the moment-map proof and can also be checked directly from (9).

5 Energy bound and global existence

Theorem 3 (Conserved quantities and complete temporal-gauge flow)**.** *For all $r \geq 1$, $a > 0$, $\kappa \geq 0$, $\mu \geq 0$ and finite initial data, equations (7)–(8) have a unique solution for every $s \in \mathbb{R}$. They preserve H, N_ψ, N_χ and every initial Gauss constraint. On the constraint surface $N_\psi = N_\chi$.*

Proof. The Hamiltonian is autonomous, so $\dot{H} = \{H, H\} = 0$. Each matter norm is conserved by its Hermitian single-particle operator, equivalently by its separate global phase symmetry. Gauss preservation was proved in Theorem 1.

The graph has maximum degree four. Unitary links and $2xy \leq x^2 + y^2$ give

$$\begin{aligned} \left|\psi^\dagger h_U \psi\right| &\leq \sum_{e=(i\leftarrow j)} 2\|\psi_i\|\|\psi_j\| \\ &\leq \sum_i \deg(i)\|\psi_i\|^2 \leq 4N_\psi. \end{aligned}$$

The same bound holds for $\chi^\dagger \bar{h}_U \chi$. Since the diamond potential and $\mu(N_\psi + N_\chi)$ are nonnegative,

$$\sum_e \operatorname{tr}(E_e^2) \leq \frac{2}{a}\left[H(0) + 4(N_\psi + N_\chi)\right]. \tag{10}$$

For fixed conserved norms, the right side is finite and nonnegative. All unitary links lie in a compact manifold; both matter vectors lie on bounded finite-dimensional spheres; Hermitian momenta are bounded by (10). The vector field is smooth on this finite-dimensional phase space. A finite-time maximal solution cannot escape every compact subset, so the continuation theorem extends it in both time directions. □

This argument does not require H to be bounded below uniformly over arbitrary matter norms. If a nonnegative Hamiltonian on the entire phase space is desired, the declared choice $\mu \geq 4$ supplies

$$H \geq \frac{a}{2}\sum_e \operatorname{tr} E_e^2 + V(U) + (\mu - 4)(N_\psi + N_\chi) \geq 0.$$

Because the two norms are conserved, the on-site term changes only species phase evolution and does not alter the link force. These completeness statements concern internal time, not physical spacetime causality or relativistic propagation.

The finite construction supplies reciprocal dynamics and constrained existence. It does not provide a continuum limit, a quantum Hilbert space of constrained states, a renormalization group, or a physical energy map. These require additional arguments rather than a reinterpretation of the finite energy bound.

6 An exact neutral seed of spatial curvature

For $r = 1$ write $U_e = e^{i\theta_e}$, $E_e \in \mathbb{R}$. Let R be the ordinary node–link incidence matrix, head $+1$, tail -1; it does not include b_e. Let C have rows $+1, +1, -1, -1$ along the two diamond paths. Then

$$RC^T = 0, \qquad \operatorname{rank} R = \operatorname{rank} C = 14, \qquad \phi = C\theta.$$

The scalar equations and constraints become

$$\dot\theta = aE, \qquad \dot E = -\kappa C^T \sin(C\theta) - (J^\psi + J^\chi), \tag{11}$$

$$\mathcal{G} = RE - |\psi|^2 + |\chi|^2 = 0. \tag{12}$$

The magnetic force has zero ordinary divergence, and the matter balance on the preceding pages gives $\dot{\mathcal{G}} = 0$ explicitly.

Corollary 4 (Neutral matter can leave the flat-link sector)**.** *Set $\theta(0) = E(0) = 0$. Let $\psi(0) = \chi(0) = c$ be real, with amplitude one at the vertex $\{0\}$ and at the edge $\{0, 1\}$, and zero elsewhere. For either diamond containing the link $e_* = \{0\} \leftarrow \{0, 1\}$, orient its curl so $C_{d,e_*} = +1$. For any $a > 0$, $\kappa \geq 0$, $\mu \geq 0$,*

$$\phi_d(s) = 2as^2 + O(s^3), \qquad 1 - \cos\phi_d(s) = 2a^2 s^4 + O(s^5). \tag{13}$$

The initial data obey all Gauss constraints, $N_\psi = N_\chi = 2$, and the solution preserves those constraints.

Proof. The two charge matrices cancel nodewise, so $\mathcal{G}(0) = 0$. At identity links all diamond gradients vanish. A matter force is nonzero only if both endpoints are occupied, selecting e_*. Increasing vertex orientation has $b_{e_*} = -1$. Formula (9) gives $\mathcal{F}^{\mathrm{m}}_{e_*}(0) = 4b_{e_*} = -4$, hence $\dot E_{e_*}(0) = 4$; all other momentum derivatives vanish. Consequently $\dot\theta(0) = 0$, $\ddot\theta(0) = 4a\,\mathbf{e}_*$. Each of the two affected diamonds has $\ddot\phi_d(0) = 4a$ in the chosen orientation. Smooth Taylor expansion and $1 - \cos x = x^2/2 + O(x^4)$ give (13). The fourth derivative of the defect is $48a^2$. Preservation follows from Theorem 1. □

Opposite initial charge densities do not imply zero current: the two species have opposite transport signs, so their charge currents add in this preparation. No external source is injected. Flatness is therefore not an invariant sector for arbitrary admissible data in this chosen model. This does not contradict the earlier theorem that a connection held flat is gauge equivalent to untwisted transport, nor turn a fixed skew operator into a curvature source without added dynamics.

7 Verification, provenance and remaining obligations

The accompanying verifier constructs B, R, C afresh and checks exact symbolic identities for scalar canonical brackets, charge-current signs, reciprocal forces, and the curvature seed. Noncommuting rank-two fixtures check matrix Gauss covariance, diamond covariance, Hermitian force gradients, local matter balance and neutral initial data. These finite fixtures support the written universal proofs; they are not exhaustive proofs for arbitrary rank and are not independent review.

A separate full-graph scalar integration uses DOP853 on $s \in [0, 1]$, $a = \kappa = 1$, $\mu = 0$, and the stated neutral seed. Its machine-readable receipt reports maximum energy drift, norm drift and Gauss residual. The early-time curvature ratio is compared with the exact Taylor coefficient. Numerical evidence is diagnostic only; no experiment or published experimental dataset enters this manuscript.

Item	Status	Remaining obligation
Joint Hamiltonian	Candidate input; conditional closure proved	Derive or select the action from source UD principles
Opposite-charge partner	Explicit extra degree of freedom	Justify matter content and preparation physically
Gauss and energy	Derived within the declared model	Independent proof review
Curvature seed	Exact Taylor result in a scalar fixture	Determine modes, continuum behavior and observables
Gauge group and rank	Arbitrary U(r) chosen	Derive physical algebra and representations
Force strengths	No physical identification	Kinetic normalization, thresholds and running
Proton and monopoles	No prediction here	Appropriate states, symmetry breaking, topology, rates

The source tetrahedral boundary, incidence graph and prescribed unitary transport family are inherited from the UD portal and the preceding *Unitary Hasse-Link Transport* v0.1 manuscript. The canonical link momenta, symplectic form, two-species matter phase space, action coefficients and Wilson-type diamond potential are declared additions. Prior frozen theorem files and bridge ledgers are not rewritten by this addendum.

The next mathematical gate is a mode and stability analysis about an admissible neutral background, with gauge directions removed. A finite U(r) label or a conserved Hamiltonian alone does not establish the Standard Model, Lorentz symmetry, a unification scale, proton decay, or magnetic monopoles.

References

[1] J. Kogut and L. Susskind, *Hamiltonian formulation of Wilson's lattice gauge theories*, Phys. Rev. D **11**, 395 (1975). doi:10.1103/PhysRevD.11.395. Primary abstract inspected; canonical lattice-Hamiltonian background.

[2] S. A. Frolov, *Gauge-invariant Hamiltonian formulation of lattice Yang–Mills theory and the Heisenberg double*, arXiv:hep-th/9501143 (1995), sec. 2, especially equations (2.11)–(2.16). https://arxiv.org/pdf/hep-th/9501143. Primary full text inspected; cotangent link variables and lattice Gauss constraints. Our Hermitian conventions are derived here rather than imported unchanged.