Unified Dynamics: Distinction, Interaction and Persistence

A theoretical account of tetrahedral carriers, information access and finite records

Benjamin Walker Mayes

15 October 2026 (UTC) — Review candidate 0.15

Contents

Abstract . . . 3

1. The problem: how does change produce things that last? 3
1.1 Object, representation and observation . . . 4
1.2 Exact representation and controlled approximation . . . 4
1.3 Scope, contribution and relation to prior work . . . 5
1.4 Assumptions and mathematical domains . . . 5

2. Distinction: specify the alternatives before explaining their evolution 6
2.1 State, observation and resolvability . . . 6
2.2 A finite binary label register . . . 7
2.3 Comparing event labels and composing a history . . . 7
2.4 A reference kernel exposes what counting cannot select . . . 7
2.5 When does continuous change count as an event? . . . 7

3. Why tetrahedral structure enters 8
3.1 Relationships before metric . . . 8
3.2 Chain conventions . . . 8

4. Interaction: what the architecture actually does 9
4.1 The declared coherent flow . . . 9
4.2 A local account of exchange . . . 9
4.3 Single-cell recurrence . . . 10

5. Symmetry: organize interactions without inventing coefficients 10
5.1 Symbolic strengths and invariant exchange sectors . . . 11
5.2 Quadratic context and its limits . . . 11

6. Shapes change function; topology changes require another rule 12
6.1 Glued spectra . . . 12
6.2 Shared interfaces and ownership . . . 12
6.3 Equal exterior boundary is insufficient for dynamical equivalence . . . 12
6.4 A transfer contract that retains the interface residual . . . 13
6.5 A family law for gluing and dynamic memory . . . 13
6.6 Exact access and reliable readout are different requirements . . . 14

7. Persistence and formation 15
7.1 State preservation does not imply attraction . . . 15

7.2 An explicit prepared growth construction 15
7.3 What a preference claim must specify 16
7.4 What the five-cell calculation adds 16

8. A distinction can survive while a readout loses access **17**
8.1 Readout error under a declared noise law 18

9. Reconstruction: what must a receipt contain? **18**
9.1 Why stocks and currents may be insufficient 18
9.2 A sufficient real receipt 19
9.3 Complex phase receipts 19
9.4 Prediction and uncertainty 19
9.5 A readable history functional also needs record capacity 20

10. From readable state to finite record apparatus **20**
10.1 Prepared history propagation 20
10.2 Timed protection and static retention differ 21
10.3 Why permanent capture needs another mechanism 22

11. Renewal, preparation and complete resource cost **22**
11.1 Renewal moves correlations 22
11.2 Coherent supply is a resource 22
11.3 What a complete cost comparison includes 23
11.4 What a prepared stationary ratio does and does not explain 23

12. Entropy, area and the physical bridge **23**

13. What the theory currently explains, and what it must explain next **24**
13.1 Reconciled native event-source boundary 24

Appendix A. Source crosswalk **25**

Appendix B. Comparison contracts **26**

Appendix C. Provenance and review boundaries **26**

Appendix D. Proofs and exact certificate conventions **27**
D.1 Skew flow and the harmonic obstruction 27
D.2 Visible closure and the retained interface 27
D.3 Orientation and reproducible spectra 28
D.4 Reconstruction and static-storage qualifications 28
D.6 Refined static joint error under an explicit input contract 28
D.7 Signed characters, channel completeness and the standard Dirac relation 29
D.8 General gluing and minimal dynamic memory 30
D.9 Conditioning, noise and sampled acquisition 32
D.10 Recurrence and transfer under explicit metric contracts 37
D.11 A prepared channel: retained coherence, fixed ratios and branching 39
D.12 Event comparisons, composable histories and winding observability 40
D.13 A declared history graph and exact receipt requirements 42
D.14 Composition of controls and the limits of phase extensions 45
D.15 A carrier-driven admission candidate with finite activity 46
D.16 Sampled acquisition and receipts of uncertain event times 49
D.17 Receipt consistency, covariance and errors it cannot reveal 51

Appendix E. Data, code and development declarations **53**

References **54**

Abstract

Finite coherent dynamics can preserve a state difference while a restricted observation loses access to it. We study this distinction on oriented tetrahedral chain carriers and separate reconstruction from finite record retention. A family-level gluing theorem shows that t tetrahedra formed by fresh-vertex face attachment have a 15t-dimensional copied state and an (8t+7)-dimensional normalized unique-simplex view. Under separate skew incidence dynamics with nonzero exchange weights in each cell, snapshot and derivative together distinguish every copied state. Any exact autonomous linear realization of the complete visible trajectory for arbitrary initial states therefore needs 7(t-1) extra state coordinates. This requirement differs from lossless residual storage for general gluings. We also derive conditioning and noise bounds, including sharp two-cell and core-star gains, a fixed-norm ill-conditioning family, and an exact sampled-readout contract. Exact spectra, observation examples and a closed-gluing control support the argument, which is integrated with conditional reconstruction, capture and renewal contracts. Standard Hodge, observability, projection-memory, synchronization and transfer methods are credited. The result concerns declared finite models: it derives neither native configuration selection nor a calibrated physical metric, universal entropy-area density or gravitational equation.

Keywords: tetrahedral complexes; incidence dynamics; persistence; projective reconstruction; finite memory; capture; renewal.

Source and scope. The technical baseline is the recovered four-paper M5 release dated 4 October 2026: Papers I–III review drafts 1.1.1 and Paper IV development draft 0.1 [1–4]. This is a separate synthesis manuscript. The four source papers retain their identities and theorem numbering. The first closure package reconciles the four paper bytes against repository commit `4b9c1f7ba03f76324b792f389517161d2bc47e05` and integrates selected later source/interface results. It is not a complete audit of every subsequent development. All results retain their stated assumptions. Review candidate; no independent review or registry promotion is claimed. Physical promotion: 0.

1. The problem: how does change produce things that last?

The world changes, yet it contains things we can recognize again. Signals travel, structures survive encounters, and interactions leave traces. A theoretical account should explain how these processes connect: how a structure is generated, how it preserves identifiable properties, and how its behaviour becomes available to another process.

These are related questions with different mathematical requirements. Existence in a state space is not a formation mechanism. Conservation of a full-state quantity is not guaranteed access through a restricted readout. A successful transfer is not permanent memory. The purpose of this manuscript is to make those distinctions part of one explanatory argument.

The organizing question is:

> Can constrained interactions explain the formation of persistent structures and readable records within a common dynamical framework?

UD investigates tetrahedral packet complexes as a candidate architecture for that framework. The word *packet* denotes a declared carrier and its state; it does not independently establish a particle, a material substance or an isolated unit of physical energy. The theory must earn each of those interpretations through a further map to observables.

The approach begins with a finite setting because its relationships and consequences can be inspected exactly. A small carrier permits an explicit boundary matrix, an exact spectrum, a complete observation map and a finite apparatus state. Larger claims should remain accountable to these constructions.

Our narrative progresses through five questions. What can differ? What can interact? What survives that interaction? What can a readout recover? What mechanism preserves the recovered difference as a record? Formation and the physical bridge then become precise obligations rather than unexplained transitions between equations.

1.1 Object, representation and observation

UD distinguishes the system being described from the mathematical model, numerical encoding, and observation used to describe it. An exact expression and a finite approximation may refer to the same intended quantity while carrying different information. Likewise, a complete state and a restricted readout can support different conclusions about distinguishability.

The programme therefore asks which relationships survive changes of representation, which distinctions remain accessible after coarse-graining, and what mechanisms make that access persistent. Numerical convenience is a methodological choice whose consequences must be accounted for. Continuity, discreteness, dimensionality, and scale invariance remain properties to be established within a justified physical description.

This is the philosophical role of distinction in the present account: it makes access and persistence explicit without equating the description with its physical referent. The methodological governance addition [10] operationalizes this separation through exact specifications, transported observables, and declared approximation errors.

1.2 Exact representation and controlled approximation

The two-tetrahedron model in §6.1 has a squared frequency 3, hence the exact frequency $\sqrt{3}$, defined by $x^2 = 3$ and $x > 0$. This is a model-derived example, not a universal physical constant. It is irrational: if $x = a/b$ in lowest terms, $a^2 = 3b^2$ gives an even exponent of the prime 3 on the left and an odd exponent on the right, a contradiction. Its nonterminating, non-eventually-periodic expansion occurs in every integer positional base. A finite expansion is rational, as is an eventually periodic expansion by a geometric-series sum. Thus changing numeral base cannot make this irrational quantity a finite fraction.

A radical must also retain its declared meaning. For example, $\sqrt{4/3} = 2/\sqrt{3}$ is distinct from $\sqrt{2}$; taking the square root of a supplied ratio does not derive a carrier frequency or physical law. A numerical match between quantities with different definitions requires an explicit correspondence. An exact expression, including $1/3 = 0.\overline{3}$, need not discard information. A finite truncation is a different object. The reduced rational a/b terminates in base B precisely when b divides B^n for some n, equivalently when all denominator primes divide B. This is an arithmetic property of encoding. It establishes no physical fractality or fundamental discreteness.

For an invertible coordinate change $y = Tx$, equivalent dynamics and observations obey

$$A_y = TA_xT^{-1}, \qquad O_y(y) = O_x(T^{-1}y).$$

If squared norm is $x^T H_x x$, its metric transforms as

$$H_y = (T^{-1})^T H_x T^{-1}.$$

Transporting the state while keeping an incompatible generator, metric or noise model changes the calculation. An information-losing projection is another operation: it needs a closure criterion and an account of the distinctions it forgets.

This discipline has a direct application to recurrence. The exact frequencies $\sqrt{3}$ and $\sqrt{5}$ have no positive common period. If operative frequencies are instead replaced by the finite decimals

$$1.732 = \frac{433}{250}, \qquad 2.236 = \frac{559}{250},$$

the rounded-frequency model has a common period 500π, since its phase advances are $2\pi \cdot 433$ and $2\pi \cdot 559$. Display rounding alone creates no such change when the exact quantities are retained internally. The artefact arises when the approximations are promoted into the dynamics.

The example does not prohibit numerical modelling. It requires the numerical model's error and intended domain to accompany its predictions. Exact algebra also remains conditional on the chosen model; it cannot establish the physical interpretation of that model by itself.

1.3 Scope, contribution and relation to prior work

The paper studies finite models of distinction, access and retention. Its reviewable claim is that these are separate mathematical obligations, illustrated by concrete tetrahedral calculations. It does not establish that tetrahedra are the unique architecture of nature. The narrative motivates the models; the declared assumptions determine their consequences.

Combinatorial Laplacians on simplicial complexes are established objects [11]. Discrete Dirac constructions likewise predate this manuscript [12]. Knill gives the complete-graph Dirac spectrum, including the filled tetrahedron [18, §9.3]. The phase equivalence in Appendix D.7 identifies its spectrum with ours. The single-cell spectrum is established background. Weighted simplicial Dirac constructions also predate this manuscript [19]. Norm conservation is a standard consequence of skew-adjointness.

Phase recovery from relative offsets belongs to the established synchronization problem [13]. The noiseless support-tree receipt in §9 gives a sufficient reconstruction under supplied nonzero-support data and component references. It is not a new general synchronization algorithm. The clock in §10 uses engineered perfect state transfer [14]; prefix dressing adds a supplied gate word, not a new origin for the clock.

The main added result is the stacked-family memory theorem in §6.5, supported by a structural rank proof. The selected glued spectra, five-cell comparison, two-cell observation obstruction and retained interface provide concrete examples. General observability and projection-memory principles are established background [20,21]. Results inherited from Papers I-IV are cited as such. The present synthesis explains how their assumptions connect and where they do not. We make no priority claim that these finite examples are absent from all prior literature; a comprehensive novelty assessment remains a submission task.

1.4 Assumptions and mathematical domains

Domain	Declared premises	Limit on interpretation
Real carrier	Finite downward-closed complex; one coordinate per nonempty simplex; increasing orientation; Euclidean simplex metric	Coordinate stocks are model quantities
Fixed flow	Constant real skew generator; no supplied switching during the interval	Full-state conservation does not imply attraction or retention
Observation	Named map, preparation alternatives, known time; noise only where stipulated	A readout is not automatically an apparatus or record
Complex receipts	Nonzero-support forest and one supplied root phase per component	Relative phases require references and acquisition
Quantum apparatus	Separate finite complex Hilbert spaces; declared joint inputs, Hamiltonians and comparator	No derivation of quantum theory from the real carrier
Resources	Count supplied donors, references, clocks, switching and exhaustion on the same domain	A subsystem energy or marginal error is not total cost
Physical bridge	No calibrated length, laboratory time, action scale or geometric metric is supplied	Physical entropy-area normalization remains open

The symbol $\mathcal{X}$ in §2.1 denotes a general state space; the binary register Θ in §2.2 is a separate label space. Model parameter s and apparatus time t are not identified. Entropies use natural logarithms and are dimensionless unless a physical conversion is separately declared.

2. Distinction: specify the alternatives before explaining their evolution

2.1 State, observation and resolvability

Let a declared state space be $\mathcal{X}$ and an observation map be $O : \mathcal{X} \to \mathcal{Y}$. Two states x, y are distinguishable under this exact observation when $O(x) \neq O(y)$. Under a noisy or finite-resolution observation, distinguishability additionally depends on the output distributions, permitted decisions and error criterion.

This operational definition makes the model's alternatives explicit. It does not imply that physical existence depends on observation, nor that every state difference is already an irreversible record. Distinction under a readout is relative to a specified map.

For real chain states, the observation might retain signed coordinates or only their squares. For complex states, it might retain amplitudes, phases or bilinear receipts. These alternatives preserve different information. A narrative that calls them all "information" without naming the map would conceal the central problem.

2.2 A finite binary label register

On four vertices, the tetrahedral edge graph has three opposite-edge matchings:

$$M_1 = \{01, 23\}, \quad M_2 = \{02, 13\}, \quad M_3 = \{03, 12\}.$$

Assigning one bit to each defines $\Theta = \mathbb{F}_2^3$. Its eight labels separate into Hamming-weight sectors of dimensions $1, 3, 3, 1$. Vertex permutations induce an onto action $S_4 \to S_3$ on the matchings, with Klein-four kernel [1, §3].

The labeled product set $S_4 \times \Theta$ has $24 \cdot 8 = 192$ elements. Under the specified reference-edge projection, its six edge fibers each have 32 elements. These are exact counts of declared objects. They do not establish a physical coupling, a 32-stage history or a clock period. On the same fiber one can supply an identity update, a transposition or a 32-cycle.

The binary register and the amplitude carrier below therefore have different roles. The first organizes labels; the second supports continuous state evolution. A source law relating them must be supplied or derived. Counting their coordinates does not provide that law.

2.3 Comparing event labels and composing a history

A receipt alphabet can distinguish event types without specifying which events can follow each other. The additional working notes [24] motivate a conditional 44-label alphabet: eight fixed-weight labels, eighteen onset labels and eighteen extinction labels, including address and parity. It has 946 unordered comparisons of distinct labels. Only 400 pairs have even one ordering with compatible incoming and outgoing Hamming weights. Full bit, frame and transport rules may reduce this set further. These counts organize possible comparisons; an admissible event graph and its update kernel remain separate obligations.

The onset/extinction block illustrates why numerical matches need typed provenance. Both its same-address subset and its reverse-weight-step subset have 108 pairs, yet their intersection contains only 36. They are different subsets. Neither their count ratio nor an assigned address cost determines a winding increment. Appendix D.12 derives the counts and shows that a winding or cost is recoverable from a receipt precisely when it is constant on each receipt fiber.

2.4 A reference kernel exposes what counting cannot select

Appendix D.13 supplies a declared fine-history model for that alphabet: a permutation of the three pair addresses followed by no flip or one addressed flip. Its eight Boolean vertices admit 192 action-distinct edges. Separating the identity/no-flip null gives 44 active receipt labels plus one null symbol. This all-open graph realizes every weight-compatible active receipt word, but its admission rule is supplied rather than derived from the continuous carrier.

A complete normalized kernel can be written on this graph. Uniform primitive controls give stationary weight populations $(1, 3, 3, 1)/8$. Other kernels with the same permutation symmetry and positive action support realize every positive stationary weight distribution. Thus a valid event alphabet and its symmetry do not determine formation preferences. The kernel's source and physical clock remain additional obligations; this is a prepared classical model, not the skew flow of §4 [25].

2.5 When does continuous change count as an event?

A changing stock does not name its own event. A response rule must specify which change matters and what persists after it. Appendix D.15 tests one explicit candidate: each opposite-edge stock drives a memory bit with distinct on and off thresholds. Once those thresholds

and an initial memory are supplied, the carrier trajectory determines a unique sequence of simultaneous update batches. The gap between thresholds gives finite activity and a bound that survives specified observation error.

The construction also reveals a constraint on the narrative. A symmetric vertex preparation can drive all three pair addresses across a threshold together. Choosing exactly one address then requires an additional reference, asymmetric state or stochastic mechanism; the coherent trajectory alone does not distinguish one. The natural covariant batch is outside the reference graph's one-flip primitive alphabet, although its endpoint belongs to the composition group. This candidate connects a carrier signal to explicit event semantics without deriving that semantics, a native geometry update or a free physical measurement [27].

Reading an event requires another contract. Finite-rate threshold polling can miss a guard touch even when the ideal response has a positive dwell time. Appendix D.16 supplies conservative sampled guards and an explicit excursion/persistence condition under which every address-wise event is confirmed within a bounded delay. Cross-address order needs additional timing separation or uncertainty receipts: one simultaneous carrier batch can split across several detection ticks. The distinction matters before a recorded order is used to compute a history-dependent quantity [29].

3. Why tetrahedral structure enters

3.1 Relationships before metric

A state list does not determine which parts interact. A simplicial complex makes those relationships explicit through containment and shared boundaries. A filled tetrahedron includes vertices, edges, faces and a cell. Its chain dimensions are

$$f(\Delta^3) = (4, 6, 4, 1), \qquad \dim \mathcal{C}(\Delta^3) = 15.$$

It is the smallest three-dimensional simplex. This supplies a compact architecture with a closed two-dimensional boundary and several incidence levels. It provides a tractable setting for exchange within a cell and coupling between cells.

This explains why it is useful to investigate tetrahedra. It does not prove uniqueness among candidate architectures or derive the dimension of physical space. A topological cell dimension, a chain-register dimension and an apparatus Hilbert-space dimension are distinct objects.

3.2 Chain conventions

Let K be a finite oriented tetrahedral simplicial complex, downward closed under faces. Each nonempty simplex occurs once. Increasing vertex order fixes the basis orientation. Set

$$\mathcal{C}(K) = \bigoplus_{k=0}^{3} C_k(K; \mathbb{R}).$$

For an oriented simplex,

$$\partial_k[v_0 \dots v_k] = \sum_{j=0}^{k} (-1)^j [v_0 \dots \widehat{v}_j \dots v_k].$$

Write B_k for its matrix. Every codimension-two face appears twice with opposite signs, giving $B_{k-1}B_k = 0$. The simplex basis is orthonormal in the present model. This is a declared Euclidean inner product on coordinates, not a derived physical spacetime metric.

The filled tetrahedron, its boundary sphere and its edge graph are different carriers. Replacing one by another changes the chain spaces, homology and operators. Their shared visual vocabulary cannot make their equations interchangeable.

4. Interaction: what the architecture actually does

4.1 The declared coherent flow

On the direct-sum chain space define [1, §4]

$$\mathcal{A}_K = \begin{pmatrix} 0 & -B_1 & 0 & 0 \\ B_1^T & 0 & -B_2 & 0 \\ 0 & B_2^T & 0 & -B_3 \\ 0 & 0 & B_3^T & 0 \end{pmatrix}, \qquad \frac{dc}{ds} = \mathcal{A}_K c.$$

The parameter s has no laboratory unit assigned. The operator is skew, so

$$\frac{d}{ds}(c^T c) = 0, \qquad e^{s\mathcal{A}_K} \in O(\mathcal{C}(K)).$$

More strongly, every pair of solutions preserves its Euclidean separation:

$$\|c(s) - \tilde{c}(s)\| = \|c(0) - \tilde{c}(0)\|.$$

This tells us what the model protects: full-state norm and distance. It does not yet tell us what a particular readout retains or how disturbances affect the model.

The chain identity also gives

$$-\mathcal{A}_K^2 = L_K, \qquad (L_K)_k = B_k^T B_k + B_{k+1} B_{k+1}^T,$$

with missing endpoint terms zero. Thus the flow's frequencies follow from the declared combinatorial Laplacian. These are conventional incidence/adjoint constructions [11,12]; the theoretical work lies in their specified carriers and consequences, not in presenting skew norm conservation as a new principle.

4.2 A local account of exchange

For adjacent simplices $\sigma \subset \tau$, define

$$J_{\sigma\tau} = 2B_{\sigma\tau} c_\sigma c_\tau.$$

The incidence contributes $-J_{\sigma\tau}$ to the lower coordinate's stock derivative and $+J_{\sigma\tau}$ to the upper coordinate's. For any coordinate set R,

$$\frac{d}{ds} \sum_{\sigma \in R} c_\sigma^2 = J_{\text{in}}(R) - J_{\text{out}}(R).$$

Internal contributions cancel. A change in the selected stock is therefore accounted for by crossing incidences. This makes "exchange" a defined balance statement. Identifying this bilinear quantity with an electrical current or physical energy flux needs an independent observation map.

4.3 Single-cell recurrence

For the filled tetrahedron, let P_0 project onto the uniform vertex mode and vanish on higher degrees. Direct boundary multiplication gives

$$\mathcal{A}^2 = -4(I - P_0), \quad \det(\lambda I - \mathcal{A}) = \lambda(\lambda^2 + 4)^7,$$

$$e^{s\mathcal{A}} = P_0 + \cos(2s)(I - P_0) + \tfrac{1}{2}\sin(2s)\mathcal{A}.$$

The single-cell operator has period π. This is a property of the chosen flow, not the identification of a universal tick of nature.

5. Symmetry: organize interactions without inventing coefficients

We use the full vertex-permutation group S_4, including odd permutations, with orientation signs on simplex chains. The standard representation T_2 is the sum-zero subspace of the four-vertex permutation representation; A_2 is the sign representation and $T_1 = T_2 \otimes A_2$. These definitions remove label ambiguity. Finite-group reduction uses standard character theory [17]. This is the signed chain action, distinct from the unsigned permutation action on six edge labels.

The natural signed S_4 action decomposes the chain carrier as [3, §2]

$$\begin{aligned} C_0 &= A_1 \oplus T_2, \quad C_1 = T_2 \oplus T_1, \\ C_2 &= T_1 \oplus A_2, \quad C_3 = A_2. \end{aligned}$$

Consequently,

$$\mathcal{C} = A_1 \oplus 2A_2 \oplus 2T_1 \oplus 2T_2.$$

Equivariance permits adjacent exchange only between common irreducible types: T_2 between vertices and edges, T_1 between edges and faces, and A_2 between faces and cell. The harmonic A_1 mode is isolated.

The real commutant has dimension $1+4+4+4 = 13$. Its skew-adjoint part has dimension three. In orthonormal multiplicity bases an equivariant skew family consists of three blocks

$$\begin{pmatrix} 0 & -\omega_r \\ \omega_r & 0 \end{pmatrix} \otimes I_{d_r}, \qquad (d_r) = (1, 3, 3),$$

plus zero on A_1. This restates the source symmetry result: three scalar channels are available. The canonical unit-weight incidence generator selects their magnitudes to be 2. Symmetry and norm conservation alone do not force that selection.

5.1 Symbolic strengths and invariant exchange sectors

Let G_k contain only the degree-$k-1/k$ exchange blocks of §4.1. On the filled tetrahedron,

$$A(a,b,c) = aG_1 + bG_2 + cG_3, \qquad a,b,c \in \mathbb{R},$$

$$\det(zI - A(a,b,c)) = z(z^2+4a^2)^3(z^2+4b^2)^3(z^2+4c^2).$$

The mutually orthogonal invariant sector projectors are $\Pi_k = -G_k^2/4$, of ranks 6, 6 and 2. Together with the harmonic rank-one projector they sum to identity. For nonzero weights the active frequencies are $2|a|, 2|b|, 2|c|$, with positive-frequency multiplicities 3, 3 and 1; a zero weight makes that entire sector stationary. Coefficient signs select the sense of rotation in an oriented channel, while the spectrum alone retains only their magnitudes. Equal frequency does not mix inequivalent irreducible channels.

The projectors follow from the incidence matrices and $B_{k-1}B_k = 0$. They organize exchange within the declared carrier; the free strengths are constitutive choices, not derived constants. The canonical unit-weight case is $a = b = c = 1$. This weighted boundary construction is standard in kind [19]; the exact convention and calculation are supplied here for review. Appendix D.7 proves the sector and character statements.

The dimensions above require full S_4. Restricting to proper tetrahedral rotations A_4 identifies the two three-dimensional types and trivializes the sign type. The full carrier becomes $3\mathbf{1} \oplus 4\mathbf{3}$, so its real commutant dimension is 25 and its skew dimension is 9. One cannot replace S_4 by A_4 while retaining the 13/3 counts. The coefficient field is also part of the model. Auxiliary finite-field modules require a declared action and an observable bridge before they can describe the real carrier. For example, the additional notes correctly exclude a surjection from the order-26 group D_{13} onto S_3; an extension by the order-four Klein group has order 104 and still cannot supply that quotient. Valid arithmetic on a chosen group or field does not select it as the native frame symmetry [24].

5.2 Quadratic context and its limits

The quadratic carrier also matters. For the standard three-dimensional representation,

$$\mathrm{Sym}^2_0(T_2) = E \oplus T_2, \qquad \mathrm{Hom}_{S_4}(E, T_2) = 0.$$

The two sectors share a quadratic parent but admit no nonzero equivariant linear identification. A context permitting $R \otimes E \to T_2$ needs $R = T_1$ or T_2 among irreducible contexts; its minimum dimension is three [3, §10]. Each permitted irreducible context has a one-dimensional intertwiner space. A three-dimensional carrier of type $A_1 \oplus E$ fails this criterion despite having the minimum numerical dimension. The representation type, not dimension alone, makes the channel available. The source constructs a compatible tensor basis, while its scalar strength remains a separate choice.

Symmetry therefore answers which interactions are possible and which are excluded. A constitutive rule must answer how strongly the permitted interactions occur. A finite symmetry group is not, by itself, a derived physical gauge group.

6. Shapes change function; topology changes require another rule

6.1 Glued spectra

Use the downward-closed complexes

$$K_A = \langle 0123, 0124\rangle, \quad K_B = \langle 0123, 0124, 0134\rangle, \quad K_C = \langle 0134, 0234, 1234\rangle.$$

Their chain dimensions are 23, 27 and 27. Exact incidence certificates give [1, §6]

$$\chi_A(\lambda) = \lambda(\lambda^2+3)^4(\lambda^2+5)^7,$$

$$\chi_B(\lambda) = \chi_C(\lambda) = \lambda(\lambda^2+2)^2(\lambda^2+5)^{11}.$$

The arrangement changes the available frequencies. Since $\sqrt{5/3}$ and $\sqrt{5/2}$ are irrational, none of these complete flows has a positive full-operator period. Selected single-frequency states remain periodic. More generally, every finite skew flow has uniformly close operator returns; Appendix D.10.1 gives a quantitative bound. Lack of a common exact period is not dissipation or irreversible history production.

6.2 Shared interfaces and ownership

In a unique-simplex complex a shared face is one coordinate, not one copy per cell. For a pure tetrahedral complex, attributing $1/n_3(\sigma)$ of each stock to each containing cell gives

$$S_\tau^{\mathrm{eq}} = \sum_{\sigma\subseteq\tau} \frac{c_\sigma^2}{n_3(\sigma)}, \qquad \sum_\tau S_\tau^{\mathrm{eq}} = \|c\|^2.$$

The equality is accounting, not a physical constitutive law. Independent copies of cells have separate coordinates and require an explicit identification map before this formula describes their assembly.

6.3 Equal exterior boundary is insufficient for dynamical equivalence

The oriented 2-to-3 move has equal exterior boundary because its difference is the boundary of [01234]. Its two sides nevertheless have different carrier dimensions and spectra. A norm-preserving coordinate inclusion need not intertwine their continuous flows.

The source also gives a concise incompatibility witness. Boundary fixation and chain compatibility force a top chain of squared norm 2 to map to a filling of squared norm 3. An isometry cannot implement that forced image [1, §6.5]. Thus boundary preservation, chain compatibility and Euclidean isometry cannot all be imposed under that linear contract.

A related inclusion has a useful positive result. The coordinate inclusion of the subcomplex K_A into K_B preserves precisely the 15-dimensional full-trajectory sector consisting of the harmonic mode and the $\sqrt{5}$ sector. The other eight dimensions are vulnerable even though the instantaneous generator defect has rank four. This is a stipulated subcomplex insertion, distinct from the boundary-fixing replacement above. Appendix D.10.2 gives the projectors, exact ranks and spectral-projection weights [23].

Subdivision also shows why the metric class belongs in a no-go statement. For the boundary-fixing 1-to-4 stellar map, unit-metric isometry fails and no positive diagonal simplex metric

gives adjoint intertwining. A general non-diagonal positive metric can do so: Appendix D.10.3 constructs one exactly. Its weighted generator is an added model, not the native unit-metric flow. Thus an unrestricted positive-metric obstruction would be false.

Geometry creation needs its own state-transfer law. Equality of boundary chains does not select it.

6.4 A transfer contract that retains the interface residual

The following finite contract and checks are developed in the first closure package [9].

Two independent tetrahedron carriers have 30 coordinates. The unique-simplex face-glued carrier has 23; the shared face contributes seven duplicate coordinates (three vertices, three edges, one face). For every consistently oriented pair of shared coordinates x,y, define

$$z = (x + y)/\sqrt{2}, \qquad r = (x - y)/\sqrt{2}.$$

Carry unshared coordinates unchanged. This defines $Q : \mathbb{R}^{30} \to \mathbb{R}^{23}$ and $R : \mathbb{R}^{30} \to \mathbb{R}^{7}$ with orthogonal completion $U = (Q; R)$, so

$$\|Qv\|^2 + \|Rv\|^2 = \|v\|^2.$$

The transfer is invertible when residuals are retained. Agreement $x = y$ sets $r = 0$ but gives $z = \sqrt{2}x$. A decoder $z/\sqrt{2}$ restores the shared amplitude convention. Simply calling z the old amplitude silently changes calibration.

This is a conditional linear interface/state-transfer contract. It does not claim chain-map compatibility or native move admission. For the direct-sum old generator, $G = U(\mathcal{A}_{\Delta^3} \oplus \mathcal{A}_{\Delta^3})U^T$, exact arithmetic gives visible/residual coupling rank 7. The visible block differs from the native glued generator with rank 14; $Q\mathcal{A}_{\text{old}} - \mathcal{A}_{\text{glued}}Q$ has rank 21. Keeping residuals closes the transformed old dynamics; discarding them does not create the native glued flow.

Choosing post-event generator $\mathcal{A}_{\text{glued}} \oplus \mathcal{A}_{\text{residual}}$ is an additional intervention, even though it preserves total norm when both blocks are skew. Its source, locality and physical cost are unclosed. This exposes the event-local generator obligation rather than hiding it inside geometric identification.

The existing subdivision results give a second contract: $T = S/\sqrt{24}$ is a bounded chain map with residual $(I - T^T T)^{1/2}$, but attenuates vertex readout and fails unweighted flow intertwining. The stricter requirement of contraction, unit singleton vertex coefficient and unweighted intertwining is already excluded by the exact 7-versus-3 incidence norm witness. These are different contracts; neither selects native geometry change.

6.5 A family law for gluing and dynamic memory

The interface problem extends beyond two cells. Keep one complete local register per tetrahedron before identifying shared simplex labels. For t cells the copied state has $n = 15t$ coordinates. Let the union have m unique simplex coordinates. For each simplex σ appearing in ν_σ copies, define

$$(Qx)_\sigma = \frac{1}{\sqrt{\nu_\sigma}} \sum_{j:\sigma\subset\tau_j} x_{j,\sigma}, \qquad QQ^T = I_m,$$

with orientation signs aligned first. An orthogonal completion adds $n-m$ residual coordinates, giving exact norm balance. These coordinates retain copied distinctions discarded by the normalized view; they are not physical memory bits or an energy estimate.

Predictive memory has a sharper definition. Let A be the copied generator and

$$d = \operatorname{rank}(Q; QA; \dots; QA^{n-1}).$$

Under exact autonomous linear evolution, linear initial encoding, arbitrary copied initial states and retention of Qx as the visible coordinates, the minimum additional state dimension is $d-m$. The observability criterion is standard [20]; Appendix D.8 gives the precise skew realization and lower bound.

Theorem 6.5 (stacked-family memory). Start from one tetrahedron and repeatedly attach a tetrahedron along one boundary face using a fresh vertex. Give each copied cell its own constant, nonzero real weights for all three adjacent-degree exchange channels. Then

$$m = 8t + 7, \qquad \operatorname{rank}(Q; QA) = 15t, \qquad h_{\min} = 7(t-1).$$

Proof idea. The newest cell has eight private coordinates containing its fresh vertex. Zero visible state and zero visible derivative force these coordinates and their derivatives to vanish. The seven attachment-face coordinates feed them through an injective block when all three weights are nonzero. The newest copy is therefore zero. Repeating backwards eliminates every copy. Appendix D.8 supplies the full argument and its limits.

For both five-cell star and chain, this yields 75 copied coordinates, 47 visible coordinates and 28 necessary hidden state coordinates. This is inherited independent-copy dynamics, distinct from the native 47-dimensional incidence law in §7.4. Snapshot/derivative injectivity explains why the apparently redundant copied information can matter to future observations. It supplies no free derivative sensor or uniform noise robustness.

The five-cell boundary of a 4-simplex is a different gluing. Under canonical copied dynamics it has $n = 75$, $m = 30$, $d = 55$: 45 coordinates retain all copied information, but only 25 extra coordinates are needed for its visible dynamics. Twenty directions remain unobservable. Thus the stacked theorem's attachment condition is substantive.

Eliminating hidden coordinates instead of retaining them gives a history-dependent visible equation with an initial-residual term. This linear projection mechanism is standard in kind [21]. Neither reduced dynamics nor a memory kernel establishes irreversible record formation. The mathematical reduction identifies the information that must be retained, encoded in a history or restricted by preparation.

6.6 Exact access and reliable readout are different requirements

The rank theorem establishes uniqueness in exact arithmetic. Noise sensitivity depends on the declared observation metric. With scale $\tau > 0$, write $M_\tau = (Q; \tau QA)$ and $H_\tau = M_\tau^T M_\tau$. If $\gamma_\tau = \sqrt{\lambda_{\min}(H_\tau)} > 0$, least-squares reconstruction gives the sharp combined-error bound

$$\|\hat{x} - x\| \le \|e\|/\gamma_\tau.$$

For canonical copies and $\tau = 1/2$, the two-cell gain is exactly $1 + \sqrt{3}$, the five-cell star gain is $\sqrt{2}(\sqrt{5} + \sqrt{3})$, and the five-cell chain gain is approximately 6.43292, with an exact rational

eigenvalue certificate. These compare the same declared copied dynamics and coordinate-noise convention, not measured physical expenditure.

If a derivative is acquired from endpoint samples separated by $2h$, endpoint errors at most ϵ give derivative error at most $\epsilon/h + \|A\|^3 Rh^2/6$ for state norm at most R. Shrinking h removes Taylor bias while amplifying noise. For canonical copies the exact sampled matrix instead uses $\sin(2h)/(2h)\, QA$, removing approximation bias. Sampling at its zeros destroys this two-block readout's active information.

Known correlated Gaussian noise is handled by $I = M^T \Sigma^{-1} M$, not by treating repeated observations as independent. Its inverse is the exact reconstruction covariance under the stated linear Gaussian model. Rescaling the observation and covariance consistently leaves I unchanged. Exact recovery can therefore coexist with poor operational performance, and changing a numerical normalization cannot manufacture information.

Appendix D.9 supplies complete proofs, uncertainty and confidence bounds, exact spectral certificates, the sampled acquisition contract and a fixed-norm counterexample to uniform robustness. Standard least-squares background is credited [22]. None supplies a physical noise law or the energy cost of sensors, references and storage.

7. Persistence and formation

7.1 State preservation does not imply attraction

Under the fixed skew flow, different initial states keep their separation. In particular, a state cannot converge to a different fixed point: its distance from that point remains constant. Distance to an invariant linear subspace is also preserved, because orthogonal evolution preserves both the subspace and its orthogonal complement.

This permits persistent modes and recurrence. It excludes attracting convergence to those targets in that particular full-state model. A projected observable can show apparent concentration without establishing full-state attraction. Open or nonlinear dynamics requires a separate analysis.

7.2 An explicit prepared growth construction

The source contains a useful assembly rule [3, §11.2]: begin with one tetrahedron and attach each new cell to a boundary face using a fresh vertex. Each step adds one vertex, three edges, three faces and one cell. It removes one boundary face and creates three. Therefore after m cells,

$$f = (m+3, 3m+3, 3m+1, m), \qquad f_{2,\partial} = 2m+2.$$

Its dual graph is a tree. Chain and star arrangements both satisfy the rule, so it supplies assembly grammar without a unique shape selector.

The earlier five-cell configuration—a core tetrahedron with one cell attached to each of its four faces—is a member of this class when all four attachments use distinct fresh vertices. It has

$$f = (8, 18, 16, 5), \quad \dim \mathcal{C} = 47, \quad f_{2,\partial} = 12.$$

These counts describe a prepared abstract complex. They do not select a physical embedding, a frequency spectrum, a formation probability or a stability advantage.

7.3 What a preference claim must specify

A structure may form rarely and survive well, or form frequently and fail quickly. Its occurrence depends on both processes. Comparing candidates requires a common initial ensemble, configuration-changing rules, state transfers, disturbances and resources.

A persistence criterion should name the retained property, tolerance and duration. Possible properties include bounded leakage, retained distinguishability or valid receipt consistency. These are different criteria. Coherent agreement explains selection only when a stated interaction makes that agreement alter formation or survival.

The source's four-coordinate averaging law provides a warning [3, §12.1]:

$$T(s) = P_0 + e^{-4s/3}(I - P_0).$$

It preserves the uniform component but suppresses contrast. Retaining total stock through averaging does not establish a localized bound structure. Formation, localization and information retention must remain separately tested.

7.4 What the five-cell calculation adds

The exact incidence certificates and comparison protocol accompany this revision [9].

The core-star has $f = (8, 18, 16, 5)$, chain dimension 47 and 12 exposed faces. Exact signed incidence construction, boundary cancellation and characteristic-polynomial factorization give

$$\begin{aligned}\chi_\star(x) = x(x^2+2)(x^2+3)^2(x^2+4)^3(x^2+5)^3(x^2+6)^2 \\ \times (x^4+9x^2+16)^3(x^4+10x^2+20)^3.\end{aligned}$$

The positive squared frequencies and each sign's multiplicities are

Squared frequency	Multiplicity of each ±i frequency
2	1
3	2
4	3
5	3
6	2
$\frac{9-\sqrt{17}}{2}$, $\frac{9+\sqrt{17}}{2}$	3 each
$5-\sqrt{5}$, $5+\sqrt{5}$	3 each

There is one zero mode. The exact norm is $\sqrt{5+\sqrt{5}}$. The squared-frequency multiplicities sum to 23, accounting for 46 active dimensions plus the zero mode.

With the stated cell ordering, the top-cell Gram matrix is

$$H_3 = \begin{pmatrix} 4 & 1 & -1 & 1 & -1 \\ 1 & 4 & 0 & 0 & 0 \\ -1 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 4 & 0 \\ -1 & 0 & 0 & 0 & 4 \end{pmatrix}.$$

Let $b = (1, -1, 1, -1)$ in the four leaf coordinates. Every leaf vector orthogonal to b has eigenvalue 4; this is a three-dimensional cell sector. On the core and normalized collective leaf $b/2$, the matrix is $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$, giving eigenvalues 2 and 6. Thus cell motion splits into three frequency-2 modes and two collective modes of frequencies $\sqrt{2}$, $\sqrt{6}$. This is an invariant top-cell oscillator decomposition, not a complete claim that all full-chain modes live on cells.

The chain's top-cell squared frequencies are $4 - \sqrt{3}, 3, 4, 5, 4 + \sqrt{3}$. Its full characteristic polynomial is recorded exactly in `results.json`; it differs from the core-star despite identical f-vectors. The full norm is approximately 2.69936488629, compared with approximately 2.68999404786 for the star before common scaling.

No complete exact period exists for the core-star because it includes frequencies $\sqrt{2}$ and $\sqrt{3}$ with irrational ratio. The frequency-2 cell sector can be periodic. Lack of a global period is not dissipation, stability or formation preference.

The numerical C2 comparison is illustrative only. A distance grid can be supplemented by the analytic Lipschitz estimate: since the generator ceiling is 1 and the initial difference norm is $\sqrt{2}$, the cell distance changes at most $\sqrt{2}$ per unit time. A nearest-grid cover gives a whole-interval error envelope from the sampled minimum. Floating evaluations have not been interval-certified, so the recorded envelope is not an exact numerical certificate. It cannot establish universal shape preference.

8. A distinction can survive while a readout loses access

Consider K_A, with two cell amplitudes $q = (q_1, q_2)^T$. Boundary cancellation gives

$$q'' = -B_3^T B_3 q = -\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} q.$$

Prepare either L with $q(0) = (1, 0)$ or R with $q(0) = (0, 1)$; set all other coordinates initially to zero, so $q'(0) = 0$. Write $c_5 = \cos(\sqrt{5}s)$ and $c_3 = \cos(\sqrt{3}s)$. Then

$$q_L(s) = \tfrac{1}{2}(c_5 + c_3, c_5 - c_3), \qquad q_R(s) = \tfrac{1}{2}(c_5 - c_3, c_5 + c_3).$$

The cell-only separation is $\sqrt{2}|c_3|$. At $s_* = \pi/(2\sqrt{3})$ the snapshots coincide. An observer receiving only those exact cell coordinates cannot recover the initial alternative; with equal priors its best error is $1/2$.

The complete state separation remains $\sqrt{2}$, and

$$q_L'(s_*) - q_R'(s_*) = -\sqrt{3}(1, -1).$$

Retaining the derivatives restores this distinction. The apparent loss belongs to the observation restriction, not to closed-state erasure.

Away from the zeros, exact noiseless cell coordinates separate these preparations perfectly. Shrinking separation becomes a gradual information-loss curve only after noise or finite resolution is specified. This example extends the source carrier calculation as an explicitly derived illustration; it does not introduce a new physical measurement claim.

More generally, a linear visible law $PA = FP$ exists exactly when $\ker P$ is invariant under A [2, Theorem II.M1]. If hidden states with equal visible outputs have different visible derivatives, no

autonomous visible law can describe both. Memory of the omitted coordinates or a larger state description is needed.

8.1 Readout error under a declared noise law

For two Gaussian output laws with known means mu_L, mu_R, common positive-definite covariance Σ and equal priors, let $\delta = \mu_L - \mu_R$. The likelihood-ratio statistic is linear, and its exact Bayes error is

$$P_e = \Phi\left(-\tfrac{1}{2}\sqrt{\delta^T \Sigma^{-1} \delta}\right).$$

Proof: center at $(\mu_L + \mu_R)/2$ and project with $\Sigma^{-1}\delta$. The score has means $\pm\delta^T\Sigma^{-1}\delta/2$ and variance $\delta^T\Sigma^{-1}\delta$; threshold zero gives the expression. At delta=0 the outputs coincide and error is 1/2. Singular covariance requires a different support-sensitive statement.

For C1 cell-only observation,

$$\delta_q = \cos(\sqrt{3}s)(1,-1), \qquad P_e(s) = \Phi\left(-\frac{|\cos(\sqrt{3}s)|}{\sqrt{2}\sigma}\right).$$

For the augmented observation, $\delta_{q'/\sqrt{3}} = -\sin(\sqrt{3}s)(1,-1)$. Consequently its separation squared is exactly 2 for every s, and

$$P_e^{\text{aug}} = \Phi(-1/(\sqrt{2}\sigma)).$$

At sigma=1/4 this is approximately 0.00233886749, while the cell-only error is 1/2 at its blind instants. The augmented receiver observes four channels rather than two, and this gain cannot be called free. No physical derivative-acquisition or retained-memory cost is closed here.

The symmetric optimal hard decision defines a binary channel with $I(Z : \widehat{Z}) = \ln 2 - h(P_e)$. This is decision information, not the full continuous-output mutual information, irreversible record entropy or information already stored in a memory. Data processing bounds it above by the full analog information.

For correlated multiple observations, stack the entire mean difference and use the complete Σ. An exact stipulated witness $\delta = (1,1)$, $\Sigma = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ gives squared Mahalanobis distance 4/3 rather than the incorrect independent value 2. Repeated observations with shared noise are not independent confirmations.

For m declared Gaussian residual coordinates with known marginal deviations sigma_i, Bonferroni gives simultaneous residual coverage at least $1-\alpha$ using thresholds $\sigma_i\Phi^{-1}(1-\alpha/(2m))$, without independence. This is coverage under the declared Gaussian law, not coverage inferred from covariance alone. Unknown covariance estimation needs its own uncertainty contract. Uncertain measurement time can mix the output laws and invalidate the known-time formula; no jitter law is assumed here.

9. Reconstruction: what must a receipt contain?

9.1 Why stocks and currents may be insufficient

Squared coefficients lose signs. On the bipartite incidence graph, adjacent products also allow opposite rescaling of even and odd degrees. If their initial norm contributions are $a, b > 0$,

rescaling by λ and λ^{-1} preserves currents and changes total norm to

$$a\lambda^2 + b\lambda^{-2}.$$

When $a \neq b$, the same original norm is obtained at both $\lambda^2 = 1$ and $\lambda^2 = b/a$. The states $(2, 1)$ and $(1, 2)$ on one incident vertex-edge pair provide a rational witness [2, §2]. Thus all adjacent products plus total norm do not generally recover the signed state.

9.2 A sufficient real receipt

Let $S = \sum c_\sigma^2 > 0$ and $\rho_\sigma = c_\sigma^2/S$. On each connected component of nonzero support choose a root and spanning tree. Supply the root sign and relative signs $\chi_{uv} = \operatorname{sgn}(c_u c_v)$ on tree edges. Propagate signs by $s_v = s_u \chi_{uv}$. Then

$$\widehat{c}_\sigma = s_\sigma \sqrt{\rho_\sigma} = c_\sigma/\sqrt{S}.$$

This reconstructs the positive ray, with explicit zeros, independently of forest choice. Root signs remain necessary because simultaneous sign reversal of a support component leaves its stocks and pair products unchanged. The receipt's sufficiency is deterministic identifiability under the supplied data, not an authentication or acquisition theorem.

9.3 Complex phase receipts

For complex coefficients define [2, §6]

$$\mathcal{J}_{\sigma\tau} = 2B_{\sigma\tau}\overline{c}_\sigma c_\tau, \qquad u_{\sigma\tau} = \frac{B_{\sigma\tau}\mathcal{J}_{\sigma\tau}}{|\mathcal{J}_{\sigma\tau}|}.$$

On nonzero support, $u_{\sigma\tau} = \overline{z}_\sigma z_\tau$, where $z_\sigma = c_\sigma/|c_\sigma|$. As in noiseless synchronization [13], supply one root phase per component and propagate forward by $z_\tau = z_\sigma u_{\sigma\tau}$, using the conjugate ratio backward. Together with normalized stocks this recovers $c/\sqrt{S}$.

The oriented cycle products must equal one. Real-part currents alone do not recover these phases: conjugation preserves stocks and real parts while generally changing the declared positive ray. References are part of the receipt's required resources.

9.4 Prediction and uncertainty

On the full-support filled tetrahedron, the incidence graph has 15 nodes and 28 edges. A frozen tree uses 14 edges and leaves 14 chords. For log amplitudes, tree observations are endpoint sums. Alternating path sums give an exact integer matrix C satisfying $H = CS$ and $z_H = Cz_T$ [2, §5].

Prediction covariance follows as $\Sigma_H = C\Sigma_T C^T$. The identity does not certify coverage without a probability model, and shared measurement noise requires residual cross-covariances. Algebraic consistency, statistical uncertainty and physical acquisition are separate claims.

The explicit split audited in Appendix D.17 sharpens this distinction. The noiseless chord residual vanishes for every latent history because H=CS, including histories generated by a wrong dynamics model. For independently acquired noisy channels, its covariance instead includes the complete training/held-out error blocks. Consistent common-field errors remain invisible, and zero residual covariance can conceal a constant bias. The residual is a receipt-consistency diagnostic; interpreting it as a prediction test requires separate state or temporal evidence [30].

9.5 A readable history functional also needs record capacity

A history receipt can identify endpoints while hiding an accumulated quantity. Appendix D.13 gives an explicit graph-cycle diagnostic whose repeated active receipt word has $N+1$ possible total increments despite known initial and final Boolean states. Exact recovery of that total needs at least $N+1$ supplemental symbols, or $\lceil \log_2(N+1) \rceil$ fixed-length bits. Merging nulls into that code increases the requirement to $2N+1$ symbols. These are conditional worst-case bounds for a declared functional and known receipt word and length, not universal physical winding laws.

The same appendix proves a complementary boundary: a scalar additive increment with full pair-permutation covariance, frame-composition consistency and flip reversal must be a difference of a function of Hamming weight. Every closed history then has zero total. The diagnostic deliberately lies outside those assumptions. Nontrivial winding needs a separately justified state and transport rule; a finite record apparatus must then acquire the required information and account for its resource cost [25].

10. From readable state to finite record apparatus

Reconstruction tells us what information is sufficient if supplied. Record formation asks what interaction produces and retains it. Paper IV makes this apparatus question explicit in finite complex Hilbert spaces [4]. This changes the mathematical description; it is not a derivation of quantum theory from real chain amplitudes.

For a declared quantum encoding and equal-prior alternatives, optimal readout error can be expressed through trace distance. Every apparatus claim must name that encoding, its joint state and its readout. We use

$$D_{\mathrm{tr}}(\rho,\sigma) = \tfrac{1}{2}\|\rho-\sigma\|_1$$

for state comparisons, and operator norm for generator mismatch. No physical unit follows from the convention $\hbar = 1$.

The apparatus includes work registers, reference, fuel, production clock, output, capture flag, records, renewal donor and retained spectators. A complete comparison includes their correlations. Good marginals cannot certify a good joint state: equal weights on $110, 101, 011$ give each bit success probability $2/3$ and zero probability of all three succeeding.

10.1 Prepared history propagation

For a supplied gate word $U_1, \ldots, U_n$, prefix dressing of the clock

$$H_C = \kappa \sum_{j=0}^{n-1} \sqrt{(j+1)(n-j)}\,(|j+1\rangle\langle j| + |j\rangle\langle j+1|)$$

uses the engineered transfer chain of [14]. Explicitly, supply $n \geq 1$, $\kappa > 0$, work generators $H_W, \bar{H}_W$, and unitary gates satisfying $[U_j, \bar{H}_W] = 0$. With prefixes $P_0 = I$, $P_j = U_j \cdots U_1$, let

$$\mathcal{W} = \sum_{j=0}^{n} |j\rangle\langle j| \otimes P_j, \quad H_p = \mathcal{W}(H_C \otimes I)\mathcal{W}^\dagger.$$

Actual and comparison generators are $I_C \otimes H_W + H_p$ and $I_C \otimes \bar{H}_W + H_p$, with $\|H_W - \bar{H}_W\| \le d$ on the full admitted domain. Prepare address 0 independently of the work/spectator state. At the endpoint the comparator applies P_n up to a common phase and the retained free evolution $e^{-it\bar{H}_W}$. Its clock norm is κn. This specifies the comparison contract of [4, Theorem IV.4]. Its endpoint probability is $\sin^{2n}(\kappa t)$, transfer occurs at $T = \pi/(2\kappa)$, and the address returns at $2T$. The prefixes and preparation are supplied; the gate-dependent address edges do not prove locality on the work registers.

If actual and comparison generators differ by at most d, Duhamel's identity gives propagator difference at most td and full-joint trace distance at most $\min\{1, td\}$. This includes the entire interval; a successful endpoint does not erase propagation error.

10.2 Timed protection and static retention differ

Timed capture supplies a terminal exchange, a fresh flag, an off-switch and a factorized guarded background. Under those premises the output is protected after pulse-off, with its stated free evolution retained [4, §5].

For timed capture, output protection follows from the factorization $H_{\mathrm{bg}} = H_O + H_{\mathrm{rest}}$, with no background interaction on output O or record M, after the supplied contacts are off. It holds for correlated actual inputs, including failures. It preserves whatever output was captured; it does not turn an incorrect capture into a correct one [4, Theorems IV.6-IV.7].

A separate static post-launch construction provides a finite window [4, Theorem IV.10]. Supply matched source/output energies, blank flag and records L, M, E, and terminal production projector P_n. Define the involution

$$J_* = P_n \otimes \mathrm{SWAP}_{SO} \otimes X_L X_M X_E + (I - P_n) \otimes I.$$

The body free generator H_f satisfies $[J_*, H_f] = 0$. A fresh independent clock Q starts at address 0, has $N + 1$ addresses ($N \ge 1$) and rate $\lambda > 0$. Dress its first edge with J_* and later edges with identity: $W_0 = I$, $W_j = J_*$ for $j \ge 1$, $\mathcal{V} = \sum_j |j\rangle\langle j| \otimes W_j$ and $H_d = \mathcal{V}(H_Q \otimes I)\mathcal{V}^\dagger$. Actual and comparison generators are $I_Q \otimes (H_f + V_p) + H_d$ and $I_Q \otimes H_f + H_d$, with $\|V_p\| \le h_p$. The complete target is the evolving clock state tensor the freely evolved body $J_* \rho J_*$, including its retained spectator. Distance to this target need not mean successful capture if the incoming production state is nonterminal.

Under these premises, the complete joint bound is

$$\epsilon(t) \le \min\{1, t h_p + 2|\cos(\lambda t)|^N\}.$$

For $\pi/(4\lambda) \le t \le 3\pi/(4\lambda)$,

$$\epsilon(t) \le \min\{1, 3\pi h_p/(4\lambda) + 2\, 2^{-N/2}\}.$$

Writing $r = |\cos(\lambda t)|^{2N}$, the comparison reduced-body distance is at most r. For incoming nonterminal probability p and fresh records, its NO_CAPTURE probability is $p + (1 - p)r$. The smaller reduced-body tail cannot replace the full joint term when clock correlations remain relevant. Appendix D.6 gives a tighter joint bound under explicitly stated fresh-record conditions. The comparator transfers exactly halfway through its cycle and undoes the transfer at a full cycle. These are pointwise bounds, not continuously monitored survival probabilities.

10.3 Why permanent capture needs another mechanism

For finite time-independent Hermitian H, an invariant subspace is reducing. Its projector commutes with H, so population cannot first enter it from its orthogonal complement. Fixed-observable expectations are finite sums of exponentials; if exactly constant on an open interval, analyticity makes them constant for all time [4, Theorem IV.9].

This excludes exact permanent first-entry storage under that closed static contract. It permits finite-window approximation, supplied switching and open-system mechanisms with their own resources. The theorem explains why protection cannot simply be inferred from successful coherent transfer.

11. Renewal, preparation and complete resource cost

11.1 Renewal moves correlations

Supply a donor D independent of the old active module A and its history Y, with matched carrier and bare energy operators. Then [4, Theorem IV.12]

$$\mathrm{SWAP}_{AD}(\rho_{AY} \otimes \sigma_D)\mathrm{SWAP}_{AD} = \sigma_A \otimes \rho_{DY}.$$

Independence means the exact product input written above, including the whole history; equal donor marginals are insufficient. For static renewal the active module includes its capture clock. The identity is kinematic; an implemented exchange requires a generator and a comparison against running contacts [4, §8.1]. The active module is refreshed; all old correlations are exported to donor and history. They are not destroyed. Repetition consumes fresh donor and output slots, and failures consume resources too.

On matching full joint domains, admitted bank error ϵ_F and operation comparisons ϵ_j compose unconditionally as

$$\epsilon_{\mathrm{total}} \le \min\{1, \epsilon_F + \sum_j \epsilon_j\}.$$

This requires uniform operation bounds on the admitted joint states. Isolated-input results do not automatically meet that requirement. The bank error is charged once; diagnostics already covered by it are not charged again as independent errors.

11.2 Coherent supply is a resource

The source constructs a uniform reference packet of length $L = 2^q$ from a seed and supplied coherent binary fuel [4, Theorem IV.2]. Here $\Delta > 0$, fuel qubit j has Hamiltonian $\Delta 2^j|1\rangle\langle 1|$ and state $|+\rangle$, and the independent reference begins at seed $|n_0\rangle$ on a sufficiently long finite ladder. The seed energy $n_0\Delta$ is additional to fuel energy. Matched disjoint exchanges transfer the supplied coherence into the reference. Ideal independent fuel has

$$\langle H_F\rangle = \Delta(L-1)/2, \qquad \mathrm{Var}\, H_F = \Delta^2(L^2-1)/12.$$

The exchanges conserve the declared sum of bare energies while transferring the supplied coherence. They do not create that coherence from an unspecified reservoir.

For a complete stationary input processed covariantly under the declared additive free-energy generators, the reference remains stationary. With nondegenerate packet energies, its distance

from the pure uniform packet is at least $1 - 1/L$ [4, Theorem IV.3]. Nonstationary supply can evade that particular obstruction, but must enter the ledger.

11.3 What a complete cost comparison includes

Cost requires declared energy operators, initial states, interaction and switching conventions, clock preparation, write and readout, retention, renewal, and finite exhaustion. Mean energy, variance, signed switching work, peak demands, time and consumed blank capacity are different quantities.

A zero signed mean switching work under one convention does not establish a free record. Thermal erasure bounds require additional reservoir and initial-state premises; finite-reservoir corrections are treated in [15]. A fresh-donor SWAP by itself supplies no thermal bath model. Nor does a matched swap establish an autonomous router. A static machine interval conserves its total energy; subsystem bare energies can change against the interaction term.

These distinctions explain why a theory of records needs the apparatus's joint state and full resource account. Without them, information-per-cost comparisons can omit the very mechanism producing the information.

11.4 What a prepared stationary ratio does and does not explain

A supplied dissipative channel can have a stable population ratio while retaining coherence much longer than a population readout suggests. Appendix D.11 examines a two-state channel motivated by the partner working notes [23]. Its stationary state is $\mathrm{diag}(4/7, 3/7)$, but its sharp trace-distance contraction per application is $(1+\sqrt{2})/\sqrt{6} \simeq 0.985599$, rather than the proposed $5/7$. Population polarization instead relaxes at magnitude $1/6$. The whole state must accompany a retained-distinguishability claim.

The ratio $4/3$ is reverse-rate divided by forward-rate for the chosen rates $2/3$ and $1/2$. An explicit third-state branch changes it to $(4/3)\alpha$. This is a conditional formation mechanism, not a derivation of a coframe ratio from incidence geometry. A channel can be dilated using a supplied environment, but its mathematical operators specify neither the environment's Hamiltonian nor renewal and switching costs. Those resources remain required before making an information-per-cost comparison.

12. Entropy, area and the physical bridge

Entropy must follow from a declared state and coarse-graining. In an illustrative symmetric binary retention channel, a uniform source bit flipped with probability p leaves a uniform memory marginal. Its marginal entropy stays $\ln 2$, while retained source information is

$$I(Z : M) = \ln 2 - h(p), \quad h(p) = -p \ln p - (1-p)\ln(1-p).$$

This elementary example is an additional operational illustration, not a thermodynamic entropy-production theorem. It shows why memory entropy alone is an inadequate record-quality measure.

A boundary-face count is a combinatorial quantity. Physical boundary area requires a geometric map and units. To derive retained information per area, a capacity or scaling mechanism must link the declared records to that boundary under complete resource constraints. None of the preceding conservation or reconstruction results supplies this automatically.

Likewise, the four-paper baseline does not derive a universal gravitational entropy density, Newton's constant or Einstein's equations. The narrative identifies what such a bridge would have to connect: finite state and records, controlled coarse-graining, physical geometry, thermodynamic quantities and independently normalized couplings. Supplying the target normalization would not constitute deriving it. Jacobson's local-horizon construction [16] assumes an entropy-area relation and a Clausius condition; citing it does not derive those inputs from the UD carrier. No Bochner-flat Kähler reduction is used or established here.

Counting events can order histories and track resources. It does not calibrate proper time. A metric, an action scale and a clock map remain additional physical obligations. This manuscript makes no claim that all these bridges are already closed.

13. What the theory currently explains, and what it must explain next

The finite account already gives a coherent explanatory sequence. Incidence specifies possible exchange. A generator calculates its consequences. Symmetry organizes channels while exposing free choices. A restricted readout can conceal a preserved distinction. Sufficient receipts restore identifiability when acquired. Prepared capture and renewal specify finite record mechanisms and their limits.

Formation remains partly supplied: boundary attachment is an explicit grammar, but the source must still determine which attachment occurs and how its state and resources transfer. A stability or occurrence preference needs the same preparation and disturbance conditions across candidates.

The next theoretical targets therefore have a dependency order:

1. Specify a native configuration-changing rule and complete state transfer.
2. Connect compatibility to formation or survival through that rule.
3. Derive a complete production-to-capture trigger and finite router on one joint domain.
4. Include reference and clock preparation, failure histories and exhaustion.
5. Establish calibrated observable and coarse-graining maps before physical identification.

Each step answers a question opened by the previous one. That is the manuscript's organizing logic: every equation earns its place by explaining a mechanism or delimiting what the mechanism can do.

13.1 Reconciled native event-source boundary

Later source audits qualify the positive interfaces and their limitations [6–8].

Later sources already provide

$$\Theta^{+} = P_g \Theta^{-} \oplus \xi$$

and an exact closure/hard-mask pushforward when the receipt xi and relevant local transport gates are supplied. The package rechecks 24,576 fixed-gate cases. This closes a conditional map, not the source of xi, gate updates or event occurrence.

At identical native pre-state, formal receipt completions xi=000 and xi=111 give different endpoints. Both identity and complementation commute with pair permutations. Hence symmetry and the fixed pre-event chain trajectory do not uniquely select the endpoint under that interface.

The positive repair must make the set of admitted next outputs a singleton, or specify a complete normalized stochastic kernel with receipt availability, null windows and timing separately typed.

The balance observable $E_3 - E_2$ supplies a pre-event signal; treating its crossing as a subdivision command is a candidate constitutive rule. The historical action/Metropolis acceptance likewise fails to select proposal frequency or admissible moves. The later polarity audit shows that universal signed-flux/index agreement and simultaneous nontrivial burden selection under binding saturation fail on stated counterexamples.

The additional receipt audit sharpens this requirement. A collection of labels with zero individual Hamming change is not itself a composable history, and the zero change does not determine an unobserved winding. To assign a winding-dependent cost, specify the fine-history domain, its admissible composition, the winding functional and what the receipt retains. Appendix D.12 gives a counterexample to inferring winding from fixed-weight labels alone. This is a source-law gap, not an argument against possible laws that supply the missing information.

The declared model in Appendix D.13 now closes a narrower development contract: it provides a complete reference kernel, a scalar no-loop theorem and sharp supplemental-receipt requirements for a specified graph-cycle diagnostic. It also proves that symmetry permits any positive stationary weight law in the kernel family. None of these statements selects native event probabilities, a physical winding or its acquisition mechanism. A positive next step is to source the additional orientation, phase, context or reservoir state and its transported increment, then test the enriched receipt fibers.

Therefore G05/G06/G09 remain source-open. The next positive task is an explicit joint state $(K, c, \Theta, \text{transport}, \text{source}, \text{reservoir})$ with lawful pre-event generator, move eligibility, transfer, generator change and resource update. Supplying one such rule establishes a candidate model; a uniqueness or native-source theorem needs additional justified restrictions. There is no derivation of that rule from the reconciled fixed-flow equations alone.

Appendix A. Source crosswalk

Manuscript topic	Four-paper source	Treatment here
Binary labels and cardinalities	I §3	Selected definitions and counting boundary
Chains, skew flow, local balance	I §§4–5	Definitions and explanatory derivations
Glued spectra and jump limits	I §6	Exact source results with assumptions
Signed contact and admission	I §§7–8	Retained as formation-source obligations; detailed contact family deferred
Real and complex reconstruction	II §§2–6	Main identifiability argument
Covariance and visible closure	II §§7–8	Selected results and scope
Symmetry and commutant	III §2	Interaction typing and scalar freedom
Confining radial refinement	III §§3–8	Outside the main argument; source retained for a later spectral chapter

Manuscript topic	Four-paper source	Treatment here
Quadratic context	III §§9–10	Symmetry obstruction and minimum context
Prepared growth and retention	III §§11–12	Formation grammar and persistence distinction
Controllers and preparation	IV §§2–4	Joint-state and supply requirements
Capture and obstruction	IV §§5–7	Timed/static distinction and finite limits
Renewal and resources	IV §§8–9	Correlation export and accounting
Two-cell observation example	I §6 plus direct derivation	New expository calculation, not physical promotion
Binary retention entropy	Declared illustrative channel	Additional operational example, not a sourced UD entropy law

This is a selective theoretical synthesis, not a concatenation or exhaustive replacement of the four papers. The radial and full contact results remain available in their source locations. No omitted source result is withdrawn by this manuscript.

Appendix B. Comparison contracts

C1 — noisy two-cell observation. Carrier is `0123,0124`, with increasing-order simplex orientations and unit-metric skew incidence flow. Source label Z is equiprobable L/R, prepared in the two unit cell states with every other coordinate zero. Observe either the two cell amplitudes, or the four coordinates $(q, q'/\sqrt{3})$. Measurement time is known. Add independent Gaussian noise of covariance `sigma² I` in the declared observation coordinates. This is a mathematical readout model, not measured/native noise. Units are model amplitude units; derivative normalization fixes an observation scale, not a free physical sensor.

C2 — five-cell architecture comparison. Compare the core-star cells `0123,1234,0235,0136,0127` with the chain `0123,1234,2345,3456,4567`. Both have 47 coordinates and 12 boundary faces. Use the same scale $\mathcal{A}/\sqrt{8}$, admitted operator-norm ceiling 1, initial norm 1, equal-prior alternatives in the first and last listed cells, cell-only readout, sigma=1/4, and horizon `[0,pi]` in the scaled model parameter. Exact rational LDL proves $8I - L$ positive definite for both, so both satisfy the ceiling. Their actual norms differ; the comparison matches a resource ceiling, not identical expenditure. Their preparation sites have different structural roles, so this single preparation does not establish architecture-wide superiority.

The reported 129-point comparison is NUMERICAL under C2. The finite horizon, noise and scale are protocol choices, not theory constants. A general worst-time or preference theorem needs additional certification or preparation averaging.

Appendix C. Provenance and review boundaries

The recovered release contains corrected editable papers, PDFs, source records and replay receipts. Its M5 corrections supersede identified passages in the preserved prior editions. This draft uses those corrected sources rather than the older reconstructed Markdown placeholders.

The four-paper release was retrieved as `UD_FOUR_PAPER_M5_v1.0_RELEASE.zip`. Its internal README dates it 4 October 2026 and states the source versions used here. A targeted first-package reconciliation checked all four source paper blobs and selected October 9–13 interface, event-source and record-entropy captures against the current repository. Uninspected developments remain outside this reconciliation. Release hashes and the selected source hashes are recorded in the companion source register.

Constructor replay is not independent review. General statements depend on their written assumptions and proofs, not on fixture counts. This draft performs editorial integration and exact checks of the two-cell and five-cell constructions, the orthogonal interface and its generator defects, and 24,576 conditional closure cases; it does not claim to rerun every historical audit.

The version-0.9 partner ingestion preserves the complete supplied working-note file and audits selected material claim families. It does not certify every imported formula or image. Important corrections include a general-metric counterexample to an overbroad subdivision obstruction and the sharp two-state contraction factor. Reference [23] identifies the source and independent replay. Constructor checks are not independent review.

The version-0.10 ingestion audits three additional partner working-note documents against the supplied v0.39 coefficient ledger and CRL-2 governance proposal. It integrates the receipt/history and cost-inference repairs, while preserving disconnected finite-group results and transport counterexamples in the companion audit [24]. Imported status labels do not constitute promotion receipts.

Version 0.11 adds a declared finite-history model and receipt-capacity analysis [25]. Its kernel and graph-cycle increment are supplied development choices. The scalar no-loop theorem and nonselection result are conditional mathematical statements, not physical promotion.

The unavailable `Gap2_FINAL_Finite_Genesis.png` was not read or used. Historical genesis fits and domain-only references are not used to substantiate physical formation, cosmological or gravitational claims. Physical promotion remains 0.

Appendix D. Proofs and exact certificate conventions

D.1 Skew flow and the harmonic obstruction

Proposition D1. Under §3.2, $\mathcal{A}^T = -\mathcal{A}$ and $-\mathcal{A}^2 = L$ with the degree blocks in §4.1. Consequently the flow preserves distances and admits no attracting fixed state with a basin containing two distinct initial states.

Proof. Transposition reverses the two adjacent blocks. Squaring leaves diagonal blocks $-(B_k^T B_k + B_{k+1} B_{k+1}^T)$; nonadjacent blocks vanish by $B_k B_{k+1} = 0$. For a difference d of solutions, $(d^T d)' = d^T(\mathcal{A}^T + \mathcal{A})d = 0$. If two distinct trajectories converged to the same fixed state, their distance would tend to zero, contradicting its positive constant value. This concerns fixed closed flow, not switched or open dynamics.

D.2 Visible closure and the retained interface

Proposition D2. Let $P : V \to W$ be a surjective linear observation and $A : V \to V$. A linear $F : W \to W$ satisfying $PA = FP$ exists if and only if $A(\ker P) \subseteq \ker P$.

Proof. Necessity follows by applying $PA = FP$ to a vector in the kernel. Conversely define $F(Px) = PAx$. If $Px = Py$, kernel invariance gives $PA(x - y) = 0$, so this definition is independent of the representative. Surjectivity makes it a linear operator on all of W. For a

nonsurjective observation the same construction is on its image, with an arbitrary extension outside that image.

Corollary D2a. For the §6.4 interface, the exact rank-seven block G_{vr} obstructs closure of the retained visible coordinates for arbitrary residual states.

Proof. In transformed coordinates the projection is $(z, r) \mapsto z$. Its kernel consists of $(0, r)$, while $G(0, r)$ has visible part $G_{vr}r$. Kernel invariance is therefore equivalent to $G_{vr} = 0$. The exact rank certificate rules this out. Retaining r closes the inherited 30-dimensional description; it does not change it into the native 23-dimensional glued dynamics.

D.3 Orientation and reproducible spectra

Proposition D3. Reversing any subset of simplex basis orientations leaves the spectrum of the unit-metric incidence generator unchanged.

Proof. Write the degree-wise sign matrices as S_k and their direct sum as S. Boundary matrices transform as $B'_k = S_{k-1}B_kS_k$. Hence $\mathcal{A}' = S\mathcal{A}S$, with $S^{-1} = S$. Characteristic polynomials and Euclidean norms are unchanged. Observations and prepared coordinates must also be transported when comparing state-dependent predictions.

The reproducible certificates use sorted simplex tuples grouped by degree and the alternating boundary formula of §3.2. Constructing the integer boundaries fixes the entire operator. Symbolic characteristic polynomials certify finite spectra; multiplicity and degree checks account for every coordinate. The star top-cell decomposition follows by splitting leaf space into $b^\perp$ and $\mathrm{span}(b)$ as in §7.4. The full-chain spectrum is independently obtained from its characteristic polynomial, not inferred from the top-cell block alone. The package stores both exact polynomials and matrices. Floating norms and the 129-point comparison remain numerical outputs.

D.4 Reconstruction and static-storage qualifications

Proposition D4. The stock and rooted-tree receipt of §9 reconstructs the normalized state on its declared support.

Proof. Each nonzero stock fixes the positive magnitude. The supplied root fixes a sign or unit phase. The unique tree path fixes every remaining sign or phase by multiplication of edge ratios, with conjugation for reversed complex edges. For receipts generated by one state, every cycle product is one, so all admissible paths agree. Without a root, multiplying a component by a common sign or phase leaves all relative receipts unchanged. This is a sufficiency result, not an optimal acquisition-cost theorem.

Proposition D5. A finite-dimensional time-independent Hermitian generator cannot produce exact permanent first-entry into an invariant subspace from its orthogonal complement.

Proof. If W is invariant, Hermiticity gives $\langle Hw, v\rangle = \langle w, Hv\rangle = 0$ for $w \in W^\perp$ and $v \in W$, so $W^\perp$ is invariant too. The projector onto W commutes with the propagator, and its population is constant. More generally, a fixed-observable expectation is a finite sum of exponential functions of time. Constancy on an open interval forces constancy everywhere by analyticity. Approximate finite-window capture, supplied time dependence and open dynamics have different premises.

D.6 Refined static joint error under an explicit input contract

Proposition D6 (conditional refinement). Use the §10.2 static comparator and a fresh independent clock at address 0. Put $r = \cos^{2N}(\lambda t)$ and $x = \mathrm{Tr}(\rho J_*) \in [-1, 1]$. Its complete body/clock/spectator trace distance from the fully transferred product target is at most

$$\sqrt{1-(1-r+rx)^2}.$$

For an arbitrary admitted body input a uniform bound is

$$g(r)=\begin{cases}2\sqrt{r(1-r)}, & 0\le r\le 1/2,\\ 1, & 1/2\le r\le 1.\end{cases}$$

If L, M, E are exactly blank and independent of the incoming body/history, then $x = p$, the incoming production nonterminal probability. In particular, a wholly terminal input ($p = 0$) gives joint error at most $\sqrt{2r-r^2}$. Actual evolution adds at most th_p to each applicable bound, capped at one. Input preparation error is a separate additive distance, not an amplitude-square term.

Proof. Purify the body and its spectator as $|\psi\rangle$. The comparator vector uses $|\psi\rangle$ at clock address 0 and $J_*|\psi\rangle$ at every other address, with common free evolution. The target uses $J_*|\psi\rangle$ at every address. Their inner product is $1 - r + rx$, since the address-0 probability is r and J_* is Hermitian. Pure-state trace distance gives the first expression; discarding a purification register contracts distance. Minimizing $|1 - r + rx|$ over $x \in [-1, 1]$ gives g. With fresh records the terminal summand flips their blank state to an orthogonal state, while the nonterminal summand is identity, so $x = p$. Lemma IV.1 supplies the actual/comparator debit.

For a pure terminal input with fresh records the joint value $\sqrt{2r-r^2}$ is exact, whereas the reduced-body distance is exactly r because the two branches have orthogonal record states. Thus the joint/marginal distinction persists even after tightening. A small distance to a target constructed from nonterminal inputs does not certify that a record was written. These bounds are an additional finite derivation in this manuscript, not an amendment to the preserved Paper IV theorem or an independent-review receipt.

D.7 Signed characters, channel completeness and the standard Dirac relation

Proposition D7. Under the signed full-S_4 action and unit simplex metric, the decomposition and commutant counts in §5 follow from the character table below. The three generator channels span the full equivariant skew family.

Conjugacy classes are ordered as identity, transposition, double transposition, 3-cycle and 4-cycle, with sizes $(1, 6, 3, 8, 6)$.

Character	Identity	(12)	(12)(34)	(123)	(1234)
A_1	1	1	1	1	1
A_2	1	-1	1	1	-1
E	2	0	2	-1	0
T_1	3	-1	-1	0	1
T_2	3	1	-1	0	-1
C_0	4	2	0	1	0
C_1	6	0	-2	0	0
C_2	4	-2	0	1	0
C_3	1	-1	1	1	-1

Proof. Signed oriented k-chains are $\Lambda^{k+1}P$, with P the four-vertex permutation representation. Traces of these exterior-power actions give the four chain rows. Taking the weighted character inner product $\langle\chi,\psi\rangle = \frac{1}{24}\sum_j n_j\chi_j\psi_j$ gives the §5 multiplicities. The five real irreducibles

have irreducible complexifications, so their real endomorphism algebra is scalar. Multiplicities $(1, 2, 2, 2)$ therefore give full, self-adjoint and skew-adjoint commutant dimensions $13, 10, 3$.

Each G_k is nonzero, skew and equivariant; their adjacent blocks have disjoint positions, so they are independent and span the three-dimensional skew family. Boundary multiplication gives $G_i G_j = 0$ for $i \neq j$, and $G_k^2 = -4\Pi_k$ with symmetric idempotent projectors of ranks $(6, 6, 2)$. The rank-one remainder is the uniform vertex mode. On each image, $G_k/2$ squares to minus identity; hence the weighted characteristic polynomial in §5.1 follows. This proves sector completeness even at zero or coincident weights.

The symmetric-square character $(\chi(g)^2+\chi(g^2))/2$ reduces to A_1+E+T_2. Likewise the multiplicity of T_2 in $E\otimes R$ is zero for $R = A_1, A_2, E$ and one for $R = T_1, T_2$. Complete reducibility extends the minimum-dimension conclusion to reducible contexts: a contributing three-dimensional irreducible summand is required.

Finally, let B be the block upper-triangular boundary operator, $D = B + B^T$ and $U = \bigoplus_{k=0}^{3} i^k I_{C_k}$. Then

$$UDU^\dagger = i(B^T - B) = i\mathcal{A}.$$

This complex unitary phase change identifies the Hermitian spectrum with the standard simplicial Dirac spectrum. It is a mathematical comparison, not a physical identification of the real carrier with a quantum state. For a filled tetrahedron it recovers the established single-cell spectrum [18].

The replay constructs all 24 signed actions, checks 576 group compositions and 72 boundary equivariance identities, and verifies the weighted polynomial and projectors exactly. It also reconstructs the Paper III §10.1 compatible context tensor and checks 72 covariance identities. These are constructor certificates for the written argument; they do not establish novelty or physical coupling strength.

D.8 General gluing and minimal dynamic memory

D.8.1 lossless normalized gluing

There is an orthogonal completion $U = (Q; R)$, with $n - m$ residual rows, such that

$$\|x\|^2 = \|Qx\|^2 + \|Rx\|^2.$$

Among linear lossless encodings retaining these m visible coordinates, at least n-m additional real coordinates are required. This is a state-dimension theorem, not a physical bit or energy cost.

Proof. Different visible rows have disjoint copied-simplex supports, and each row has squared norm $\nu_\sigma/\nu_\sigma = 1$. Complete the rows by an orthonormal basis of $\ker Q$. Orthogonality proves the identity. An injective linear encoding of an n-dimensional state has rank n, so m+r output coordinates require $r \geq n - m$. If the copies all equal c_σ on a simplex, $Qx = \sqrt{\nu_\sigma} c_\sigma$; recovering the native coordinate requires division by $\sqrt{\nu_\sigma}$. Native and copied norm conventions differ.

D.8.2 minimal autonomous linear memory

For any real skew A on $\mathbb{R}^n$ and Q with m orthonormal rows, define

$$\mathcal{O} = (Q; QA; \ldots; QA^{n-1}), \qquad d = \operatorname{rank} \mathcal{O}.$$

To reproduce $Qe^{sA}x_0$ exactly for every x_0 and $s \geq 0$ using a time-independent autonomous linear state model, initialized by a linear encoding of x_0 and retaining Qx as its first m coordinates, the minimum number of additional state coordinates is $d - m$.

Proof. The observable subspace $W = \operatorname{im} \mathcal{O}^T$ is A-invariant by Cayley-Hamilton and skewness. Its orthogonal complement is also invariant. The subspace W contains $\operatorname{im} Q^T$. Choose an orthonormal basis for W starting with the rows of Q and extend them by d-m rows H in W intersect $\ker Q$. With $T = (Q; H)$, $F = TAT^T$ is skew and $TA = FT$, so the d-dimensional model reproduces the visible trajectory exactly. Conversely, any linear initial encoding C supporting exact future outputs has $\ker C \subseteq \ker \mathcal{O}$: a state encoded as zero must have every visible derivative zero. Hence $\operatorname{rank} C \geq d$ and at least d-m extra coordinates are required. This lower bound concerns exact finite autonomous linear realizations for arbitrary initial states; it is not a bound for one fixed preparation, a nonlinear encoding or an apparatus with external stored history.

The underlying observability construction is established control theory. The skew case permits an orthogonal reducing realization. The gluing-specific contribution is the concrete observation, its dimension and the structural rank argument below. No priority claim is made for the general observability principle.

D.8.3 fresh-vertex stacked tetrahedra require all residuals

Start with one tetrahedron and attach each later tetrahedron along one boundary face using a new vertex. In each copied cell let all three adjacent-degree weights a_j, b_j, c_j be nonzero real constants; they need not agree between cells.

Then

$$n = 15t, \qquad m = 8t + 7, \qquad \operatorname{rank}(Q; QA) = 15t.$$

Consequently $d = n$, and exactly $7(t - 1)$ additional coordinates are required by §D.8.2. The visible/residual derivative coupling has rank $7(t - 1)$. For $t = 1$ this count is zero.

Proof. Attachment adds 1 vertex, 3 edges, 3 faces and 1 cell, so the unique register grows by 8 from its initial 15 coordinates. Consider the last attached tetrahedron, with fresh vertex v. Its eight simplex coordinates containing v occur in no earlier cell. If $Qx = 0$ they vanish in this last copy. If $QAx = 0$ their local derivatives also vanish, because the copied generator is block diagonal.

The remaining seven coordinates belong to the attachment face. The map from these face coordinates to derivatives of the eight private coordinates has Gram matrix

$$C^T C = \operatorname{diag}(a_j^2, a_j^2, a_j^2, b_j^2, b_j^2, b_j^2, c_j^2).$$

Indeed each face vertex contributes through its new edge to v, each face edge through its new face containing v, and the attachment face through the new cell; orientation signs disappear in the Gram matrix. Nonzero weights make this map injective. Thus the last copied cell is zero. Remove it and repeat backwards: contributions of already-zero later copies vanish in both Qx and QAx, and every remaining row normalization is nonzero. Finally the original copy is zero. Therefore $\ker Q \cap \ker(QA) = \{0\}$, proving the rank claim. On $\ker Q$, QA is injective, so its rank is $n - m$, which is the visible/residual coupling rank under any orthogonal residual basis.

This proves a family result, not just a rank observation on the two-cell fixture. It also identifies the hypotheses: fresh-vertex attachment, separate copied generators, nonzero exchange weights,

and unrestricted copied initial states. Cyclic gluing or a zero channel need not satisfy the conclusion.

D.8.4 Exact projected dynamics

In orthogonal visible/residual coordinates, write UAU^T as blocks $G_{vv}, G_{vr}, G_{rv}, G_{rr}$. Eliminating the residual gives

$$z'(s) = G_{vv}z(s) + G_{vr}e^{sG_{rr}}r_0 + \int_0^s G_{vr}e^{(s-u)G_{rr}}G_{rv}z(u)\,du.$$

This follows directly from variation of constants in $r' = G_{rv}z + G_{rr}r$. The initial residual term cannot be discarded for arbitrary preparation. The kernel is a deterministic memory term; it is not automatically physical noise or dissipation. Skewness gives $G_{rv} = -G_{vr}^T$, so its instantaneous value is $-G_{vr}G_{vr}^T$, but its time dependence and initial term retain the underlying reversible dynamics. This is the linear projection/memory mechanism familiar from Mori-Zwanzig reduction, not a new irreversible entropy law.

D.9 Conditioning, noise and sampled acquisition

D.9.1 Declared readout and its conditioning

Use the copied-state generator $A^T = -A$ and normalized gluing observation Q, with $QQ^T = I_m$, from the stacked-family theorem. The copied state has n real coordinates. Declare an observation scale $\tau > 0$ and

$$y = M_\tau x + e, \qquad M_\tau = \begin{pmatrix} Q \\ \tau QA \end{pmatrix}, \quad e = \begin{pmatrix} e_0 \\ \tau e_1 \end{pmatrix}.$$

Here e_1 is raw derivative error; its acquisition is not assumed free. The scale makes the two blocks commensurate in the chosen model units. It is not a new dynamical coefficient or a physical clock calibration. Assume full column rank when reconstructing the complete state.

Define

$$H_\tau = M_\tau^T M_\tau = P + \tau^2 A^T P A, \quad P = Q^T Q, \quad \gamma_\tau = \sqrt{\lambda_{\min}(H_\tau)} > 0.$$

The absolute noise gain is $1/\gamma_\tau$. The relative matrix condition number is $\kappa_2(M_\tau) = \sigma_{\max}(M_\tau)/\gamma_\tau$. These are different quantities.

Theorem C1 (sharp deterministic error). The least-squares reconstruction $\hat{x} = M_\tau^\dagger y$ satisfies

$$\boxed{\|\hat{x} - x\| \le \frac{\sqrt{\epsilon_0^2 + \tau^2\epsilon_1^2}}{\gamma_\tau}} \quad \text{if } \|e_0\| \le \epsilon_0,\ \|e_1\| \le \epsilon_1.$$

As a bound in the combined error norm $\|e\|$, the factor $1/\gamma_\tau$ is attained by error along a least singular left vector. Separate block budgets need not attain this combined bound simultaneously.

Proof. Full rank gives $M_\tau^\dagger M_\tau = I$. Hence $\hat{x} - x = M_\tau^\dagger e$, and the pseudoinverse norm is $1/\gamma_\tau$. A singular-vector error attains the norm. This is standard SVD-based least-squares conditioning, specialized to the declared observation.

If M_τ is rank deficient, no finite bound recovers arbitrary full states even at zero noise. The pseudoinverse recovers only the projection onto its row space. That space need not equal the full time-observable subspace when higher derivatives are required.

D.9.2 Generator uncertainty and usable certificates

Let $\bar{A}$ be the reconstruction model, $\|A - \bar{A}\| \le \delta$, with the same exact Q. Set $\bar{M}_\tau = (Q; \tau Q\bar{A})$ and $\bar{\gamma}_\tau = \sigma_{\min}(\bar{M}_\tau)$. For $\|x\| \le R$,

$$\boxed{\|\bar{M}_\tau^\dagger y - x\| \le \frac{\sqrt{\epsilon_0^2 + \tau^2\epsilon_1^2} + \tau\delta R}{\bar{\gamma}_\tau}.}$$

This assumes $\bar{\gamma}_\tau > 0$. It owns model mismatch separately from measurement noise. Since $\|M_\tau - \bar{M}_\tau\| \le \tau\delta$, minimization over unit vectors gives

$$|\gamma_\tau - \bar{\gamma}_\tau| \le \tau\delta.$$

Thus a certified true lower bound $\gamma_\tau \ge \Gamma$ guarantees reconstruction-model injectivity when $\tau\delta < \Gamma$, with denominator at least $\Gamma - \tau\delta$.

Proof. Write $y = \bar{M}_\tau x + e + (M_\tau - \bar{M}_\tau)x$ and apply C1. The singular-value inequality follows from $\|\bar{M}v\| \ge \|Mv\| - \|(\bar{M} - M)v\|$ for unit v, and then interchange the matrices. No small-error expansion is used.

For exact rational H_τ, a rational LDL factorization of $H_\tau - qI$ with positive pivots certifies $\gamma_\tau > \sqrt{q}$. Floating eigenvalues may suggest q but do not certify it. A general, usually conservative, bound is

$$\gamma_\tau \ge \frac{\sqrt{\det H_\tau}}{(1 + \tau^2\|A\|^2)^{(n-1)/2}},$$

because every Gram eigenvalue is at most $1 + \tau^2\|A\|^2$. Positivity of the determinant is available only after full rank is established. Unknown errors in Q require an additional observation-model term and are not included in δ.

D.9.3 Gaussian noise, correlations and representation invariance

Assume the complete error e is zero-mean Gaussian with known positive-definite covariance Σ, independent of the unknown fixed state. The generalized least-squares estimate and information matrix are

$$\hat{x} = I_\Sigma^{-1} M_\tau^T \Sigma^{-1} y, \qquad I_\Sigma = M_\tau^T \Sigma^{-1} M_\tau.$$

Theorem C2 (exact covariance and confidence region). Under full column rank and the exact measurement model,

$$\mathrm{Cov}(\hat{x} - x) = I_\Sigma^{-1}, \qquad \mathbb{E}\|\hat{x} - x\|^2 = \mathrm{tr}(I_\Sigma^{-1}),$$

$$\Pr\big[(\hat{x} - x)^T I_\Sigma (\hat{x} - x) \le \chi^2_{n,1-\alpha}\big] = 1 - \alpha.$$

In particular, a Euclidean confidence radius is

$$\|\hat{x} - x\| \le \sqrt{\chi^2_{n,1-\alpha}/\lambda_{\min}(I_\Sigma)}.$$

Proof. The estimator error is the displayed linear map applied to e. Multiplying its covariance gives I_Σ^{-1}. Whiten this n-dimensional Gaussian error to obtain the chi-square statement. The radius follows from the smallest information eigenvalue. Gaussianity is needed for this exact coverage, not for the covariance identity. Bias, covariance estimation and non-Gaussian tails need separate bounds.

For equal-prior alternatives separated by Δx under the same covariance, the exact optimal decision error is

$$P_e = \Phi\Big(-\tfrac{1}{2}\sqrt{\Delta x^T I_\Sigma \Delta x}\Big) \le \Phi\Big(-\tfrac{1}{2}\sqrt{\lambda_{\min}(I_\Sigma)}\|\Delta x\|\Big).$$

This follows by the likelihood-ratio projection onto $\Sigma^{-1}M_\tau\Delta x$. It is distinguishability of the readout distributions, not information already written into a retained physical memory.

For an invertible output rescaling S, transport all three objects:

$$y \mapsto Sy, \quad M \mapsto SM, \quad \Sigma \mapsto S\Sigma S^T.$$

Then I_Σ is unchanged. Increasing τ on paper cannot improve statistical information while leaving the raw noise physics fixed. An unweighted singular value can change under this rescaling; a consistent whitened calculation cannot. Correlated observations require the full covariance, including cross-block terms.

D.9.4 Derivative acquisition from three snapshots

Observe y_-, y_0, y_+ at $s_0 - h, s_0, s_0 + h$, with $h > 0$ and $s_0 \ge h$. Prepare the state and retain its history under the same constant generator throughout. Define

$$D_h = \frac{y_+ - y_-}{2h}.$$

If each endpoint has norm error at most ϵ, $\|A\| \le L$ and $\|x(s_0)\| \le R$, then

$$\boxed{\|D_h - QAx(s_0)\| \le \frac{\epsilon}{h} + \frac{L^3 R h^2}{6}.}$$

Proof. Endpoint errors contribute at most $2\epsilon/(2h)$. The third derivative of Qx is QA^3x, whose norm is at most L^3R by skew-flow norm preservation. The centered Taylor remainder gives the second term. No noiseless derivative is assumed.

Finite precision and timing fit the same budget. Suppose each m-coordinate snapshot has additional detector error at most ϵ_{det}, componentwise rounding to nearest with step q and no saturation, and timestamp error at most j. Its total deterministic error is at most

$$\epsilon_{\text{eff}} = \epsilon_{\text{det}} + \frac{\sqrt{m}\,q}{2} + LRj.$$

The rounding vector has at most $q/2$ in each component, and integrating $\|Qx'\| \le LR$ over the timing offset gives the last term. Use the appropriate effective budget for each center/endpoint in C1 and the difference formula. Correlated or stochastic timing errors are not thereby independent Gaussian noise.

Substitute this derivative error into C1. For this conservative derivative-approximation bound alone, the unconstrained minimizing step is

$$h_* = (3\epsilon/(L^3R))^{1/3},$$

when $L, R, \epsilon > 0$. Actual latency or admissible-time restrictions may exclude it. This is a minimum of the bound, not a universal sampling optimum.

For independent endpoint Gaussian errors with covariance $\sigma^2 I_m$, derivative-noise covariance is $\sigma^2 I_m/(2h^2)$. With an independent center snapshot of the same noise, the cross covariance with y_0 is zero. The derivative mean-square error obeys

$$\mathbb{E}\|D_h - QAx\|^2 \le \frac{m\sigma^2}{2h^2} + \frac{L^6R^2h^4}{36}.$$

Its unconstrained bound-minimizing step satisfies $h_*^6 = 9m\sigma^2/(L^6R^2)$. Shared temporal noise changes these formulas. A three-snapshot acquisition uses $3m$ scalar readout channels over a span $2h$, plus clock alignment, bandwidth and storage resources; this is not a complete energy cost.

D.9.5 Exact canonical sampling avoids truncation bias

For the canonical independent-copy generator, $A^3 = -4A$. Consequently

$$\frac{e^{hA} - e^{-hA}}{2h} = f(h)A, \qquad f(h) = \frac{\sin(2h)}{2h}.$$

This can be used as the exact measurement matrix rather than approximating the derivative. The retained statistic (y_0, D_h) has

$$M_h = \begin{pmatrix} Q \\ f(h)QA \end{pmatrix}, \qquad \Sigma_h = \sigma^2 \operatorname{diag}(I_m, I_m/(2h^2))$$

under the independent equal-noise three-snapshot contract. Its information matrix is exactly

$$\boxed{I_h = \sigma^{-2}\left[P + \frac{\sin^2(2h)}{2}A^TPA\right].}$$

Thus $h = \pi/4$ maximizes this information matrix in Loewner order over $0 < h < \pi/2$. It is a model-unit optimum for this retained statistic under fixed snapshot noise; it is not an optimum over every sensor, time budget or the full three-snapshot record. The symmetric endpoint sum has been discarded. At $h = k\pi/2$, $k \ne 0$, the derivative statistic loses its active signal, so this two-block readout is rank deficient for a nontrivial stack.

Proof. On harmonic modes both sides of the first identity vanish; on every active mode A has frequency 2, giving the sine factor. The covariance inverse contributes $2h^2/\sigma^2$, which cancels the factor $1/(4h^2)$ in $f(h)^2$. The remaining coefficient is maximized when $\sin^2(2h) = 1$. At its

zeros, only the snapshot remains. Calibrating the exact matrix removes Taylor bias, not noise amplification or acquisition cost.

D.9.6 Exact two-cell and five-cell conditioning

The following values concern independent copied dynamics and the normalized union observation, with $\tau = 1/2$ and canonical weights. They are not the native union-generator frequencies. All copied generators have norm 2.

Configuration	n	m	Minimum Gram eigenvalue	Absolute error gain $1/\gamma$
Two cells, shared face	30	23	$1-\sqrt{3}/2$	$1+\sqrt{3} \approx 2.73205$
Five-cell core-star	75	47	$1-\sqrt{15}/4$	$\sqrt{2}(\sqrt{5}+\sqrt{3}) \approx 5.61177$
Five-cell chain	75	47	Smallest root of $64z^4 - 256z^3 + 280z^2 - 48z + 1$	Approximately 6.43292

For the chain, exact LDL and Sturm checks place the minimum Gram eigenvalue in $(241/10000, 243/10000)$. Hence an entirely rational certificate gives error gain below $100/\sqrt{241}$. The decimal estimate is more precise but is not itself the certificate. All three maximum Gram eigenvalues are 2, so their relative condition numbers are $\sqrt{2}$ times the listed absolute gains.

Proof for two cells. The exact Gram polynomial factors as

$$\frac{(z-2)^{14}(z-1)(4z-7)(4z^2-8z+1)^7}{2^{16}}.$$

Its smallest root is $1-\sqrt{3}/2 = ((\sqrt{3}-1)/2)^2$. Inverting its square root gives $1+\sqrt{3}$. The star's exact factorization, stored in the certificate, has its smallest root in $16z^2-32z+1$. All its other factor roots are at least this value. For the chain, the full characteristic polynomial's square-free part has exactly one root in the certified interval and none between zero and its lower endpoint; that root belongs to the displayed quartic. These statements are checked in exact arithmetic.

For the same combined output error budget 0.01, worst-case state errors are at most approximately 0.02732, 0.05612 and 0.06433 respectively. These are worst-direction readout comparisons. They do not establish universal architecture superiority or equal physical expenditure.

D.9.7 Exact recoverability has no uniform robustness guarantee

Even keeping the copied-generator norm equal to 2 does not give a positive uniform γ over all nonzero weights. In the two-cell example set $a = b = 1$, $c = \varepsilon > 0$ in each cell. Let v be the unit difference of their shared-face coordinates. Then

$$Qv = 0, \qquad \|QAv\| = \varepsilon, \qquad \gamma_\tau \le \tau\varepsilon.$$

Therefore the snapshot/derivative noise gain is at least $1/(\tau\varepsilon)$ and diverges as $\varepsilon \downarrow 0$, although exact rank remains full for every positive ε. The other channels keep $\|A\| = 2$. At $c = 0$ a higher derivative can restore time observability, so this is specifically a failure of uniform robustness of the declared two-block readout, not a theorem that all possible observation histories fail.

For full-state reconstruction on the closed five-cell boundary, the two-block matrix has a 20-dimensional kernel even before noise. Restricting to its certified observable subspace permits positive singular-value bounds there; adding a finite noise budget cannot identify the discarded directions.

D.9.8 Scope, verification and references

The pseudoinverse, whitening and Gaussian arguments are standard statistical linear algebra. The specialized results here are their application to the declared gluing observation, its exact spectra, the fixed-norm degenerating family and the exact canonical sampling contract. Constructor replay supplies exact characteristic-polynomial/LDL/Sturm certificates plus separately labelled floating fixtures. It does not estimate a physical noise covariance, establish independent review, certify measured bandwidth or prove a physical entropy-area relation.

Primary background: LAPACK *Users' Guide*, Linear Least Squares Problems, and How to Measure Errors. General perturbation literature includes G. W. Stewart, On the Perturbation of Pseudo-Inverses, Projections and Linear Least Squares Problems; only its abstract/metadata were inspected, so no specific theorem from its paywalled text is invoked here. The displayed bounds have self-contained proofs.

Next requirements for operational use are calibrated joint covariance, acquisition and clock-error models, admissible sampling windows, generator uncertainty and a complete resource ledger. The derivation closes a mathematical conditioning gap while leaving those physical inputs explicit.

D.10 Recurrence and transfer under explicit metric contracts

D.10.1 Incommensurability permits uniformly close returns

Let A be a finite real skew matrix with $r \geq 1$ distinct positive frequencies $\omega_1, \dots, \omega_r$. For every integer $N \geq 1$ there is $1 \leq q \leq N^{r-1}$ such that, at $s = 2\pi q/\omega_1$,

$$\|e^{sA} - I\|_2 \leq \frac{2\pi}{N}.$$

For $r > 1$, partition the $(r-1)$-torus into N^{r-1} cubes of side $1/N$. Among the $N^{r-1}+1$ points with coordinates $\{j\omega_k/\omega_1\}$, $j = 0, \dots, N^{r-1}$ and $k = 2, \dots, r$, two share a cube. Their index difference is the required q, with each phase within $2\pi/N$ of an integer multiple of 2π. Orthogonal block diagonalization of A reduces the operator error to the largest rotation-block error, bounded by the phase error. Zero modes contribute zero error. For $r = 1$, choose $q = 1$ and the return is exact. If $r = 0$, the flow is the identity.

Consequently, irrational frequency ratios exclude an exact common period but do not exclude approximate recurrence. If there is no exact period, the approximating times have an unbounded subsequence; otherwise a bounded subsequence would yield an exact return as $N \to \infty$. In the periodic case, arbitrarily late exact returns exist. This is a finite-flow statement; no thermodynamic limit, mixing, dissipation or record formation follows from it.

D.10.2 A protected sector for a specified inclusion

Use the complexes K_A, K_B of §6.1 and the coordinate inclusion $T : \mathbb{R}^{23} \to \mathbb{R}^{27}$ that fixes every simplex of K_A. This is an inclusion of subcomplexes, not a boundary-fixing 2-to-3 replacement. Write $A = \mathcal{A}_{K_A}$, $B = \mathcal{A}_{K_B}$ and $L = -A^2$. With unit metrics,

$$T^T T = I, \quad d_B T = T d_A, \quad R = BT - TA,$$

where d is the total boundary. Exact signed incidence gives

$$\operatorname{rank} R = 4, \quad RR^T R = 2R, \quad \operatorname{rank} \begin{pmatrix} R \\ RA \end{pmatrix} = 8.$$

Thus the four nonzero singular values of R are $\sqrt{2}$. The polynomial projectors are

$$P_0 = \frac{(L-3I)(L-5I)}{15}, \quad P_3 = \frac{L(5I-L)}{6}, \quad P_5 = \frac{L(L-3I)}{10}.$$

They are mutually orthogonal and sum to I. Direct incidence identities give $R(P_0 + P_5) = 0$ and $\operatorname{rank}(P_0 + P_5) = 15$. Since this projector commutes with A, its image lies in the kernel of every RA^j. The displayed rank-eight two-block matrix shows that no larger protected space exists. Therefore

$$\bigcap_{j \geq 0} \ker(RA^j) = \operatorname{im}(P_0 + P_5), \qquad e^{sB} T x = T e^{sA} x$$

for precisely these initial states. A zero instantaneous residual alone would give a larger, insufficient kernel.

The remaining eight-dimensional sector admits a more detailed description. Put $L_B = -B^2$ and $P_2^B = L_B(5I - L_B)/6$. The positive compression

$$W = P_3 T^T P_2^B T P_3$$

has rank four and satisfies $W^2 = (2/3)W$. On $\operatorname{im} P_3$ its eigenvalues are $2/3$ and 0, each fourfold. Corresponding normalized states have squared spectral-projection weights $(2/3, 1/3)$ or $(0, 1)$ in the $L_B = 2, 5$ sectors. These are instantaneous spectral weights after a stipulated inclusion, not transmission probabilities of a derived scattering experiment. Their exact matrices and ranks are replayed in [23].

D.10.3 Stellar subdivision: diagonal obstruction and a general-metric counterexample

Subdivide [0123] using interior vertex 4 into $0124, 0134, 0234, 1234$. Let d_c, d_f be the coarse and fine total boundaries. The map S fixes the old boundary and sends the oriented top cell to

$$S[0123] = [0124] - [0134] + [0234] - [1234].$$

Then $d_f S = S d_c$ and $S^T S = I_{14} \oplus [4]$. Boundary fixation and the unique filling force this image: the fine d_3 is injective. Hence this map cannot be an isometry for unit simplex metrics.

For a positive *diagonal* fine metric H, the adjoint is $d_f^{\dagger_H} = H^{-1} d_f^T H$. Adjoint intertwining on the old four boundary faces would require a matrix with four separate nonzero face-to-cell entries to equal the rank-one matrix $S_3 d_{c,3}^T$. Positive diagonal weights cannot change these ranks. Thus diagonal weighting alone cannot provide this stronger intertwining contract.

That argument does not exclude all positive metrics. Define the simplicial chain retraction Π by sending vertex 4 to 0 and fixing 0,1,2,3; a simplex with repeated image vertices maps to zero, and other images carry their permutation sign. It obeys

$$\Pi S = I, \qquad \Pi d_f = d_c \Pi.$$

Put $N = I - S\Pi$ and choose the degree-preserving, generally non-diagonal metric

$$H = \Pi^T \Pi + N^T N.$$

For any v, $v^T H v = \|\Pi v\|^2 + \|Nv\|^2 > 0$ unless $v = 0$, since $v = S\Pi v + Nv$. Also $S^T H S = I$. The decomposition $\operatorname{im} S \oplus \ker \Pi$ is H-orthogonal and each summand is d_f-invariant. Therefore the weighted adjoint preserves the same summands and restricts on $\operatorname{im} S$ to the coarse Euclidean adjoint. It follows that

$$A_H = H^{-1} d_f^T H - d_f, \qquad A_H S = S(d_c^T - d_c), \qquad A_H^T H + H A_H = 0.$$

This explicitly refutes an unrestricted positive-metric no-go. It does not supply unit-metric intertwining or a native choice of H. The weighted generator is an added model, with correlated simplex coordinates; its locality and physical admissibility require a separate contract. The supplied certificate verifies all identities exactly and gives positive rational LDL pivots [23].

D.11 A prepared channel: retained coherence, fixed ratios and branching

The following operators are declared mathematical channels, motivated by the partner working notes [23]. They are not derived from a UD carrier-to-bath interaction. For a two-state Hilbert space, take

$$K_0 = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{3} \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & \sqrt{2/3} \\ 1/\sqrt{2} & 0 \end{pmatrix}, \qquad \mathcal{E}(\rho) = \sum_{j=0}^{1} K_j \rho K_j^\dagger.$$

Completeness is $\sum_j K_j^\dagger K_j = I$. In Bloch coordinates $\rho = (I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z)/2$, direct multiplication gives

$$r_x' = a_x r_x, \quad r_y' = a_y r_y, \quad r_z' = -r_z/6 + 1/6, \qquad a_x = \frac{1+\sqrt{2}}{\sqrt{6}}, \quad a_y = \frac{1-\sqrt{2}}{\sqrt{6}}.$$

All linear eigenvalues have modulus less than one, so the unique fixed point is $\rho_* = \operatorname{diag}(4/7, 3/7)$. The repeated channel has $r_x(k) = a_x^k r_x(0)$, $r_y(k) = a_y^k r_y(0)$ and $r_z(k) - 1/7 = (-1/6)^k (r_z(0) - 1/7)$. Coherences decay asymptotically; they are not erased in one step.

For qubit states, trace distance is half the Euclidean distance of their Bloch vectors. The sharp one-step contraction coefficient, and its k-step counterpart, are consequently

$$\eta(\mathcal{E}) = a_x \simeq 0.985599, \qquad \eta(\mathcal{E}^k) = a_x^k.$$

Opposite σ_x eigenstates attain the bound. The same sharp factor applies to distance from ρ_* by taking sufficiently small x-axis perturbations around that full-rank state. Thus $(5/7)^k$ is not a valid universal bound for this channel. An additional projection could change a contracted readout, but requires its own defined action and comparison domain.

The population transition rates are $u = 1/2$ from state 1 to 2 and $v = 2/3$ from 2 to 1. For arbitrary $0 < u, v < 1$, the same construction with $K_0 = \mathrm{diag}(\sqrt{1-u}, \sqrt{1-v})$ and off-diagonal K_1 yields stationary populations $(v, u)/(u + v)$. Therefore $p_1^*/p_2^* = v/u$. The value 4/3 follows from the chosen rates; its numerical agreement with a separately stated geometric ratio does not derive those rates or identify the populations with that geometry. At some other rates the population fixed point need not be the unique full-state fixed point; uniqueness above was proved for the displayed channel.

A three-state extension makes the dependence explicit. For $0 \le \alpha \le 1$, let

$$K_0 = \mathrm{diag}(1/\sqrt{2}, 1/\sqrt{3}, 1/\sqrt{7}),$$

$$K_1 = \begin{pmatrix} 0 & \sqrt{2\alpha/3} & 0 \\ 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad K_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{6/7} \\ 0 & \sqrt{2(1-\alpha)/3} & 0 \end{pmatrix}.$$

Their products sum to I_3 and diagonal states remain diagonal. Their stationary population vector is

$$p_* = \frac{(12\alpha, 9, 7(1-\alpha))}{16 + 5\alpha}, \qquad \frac{p_1^*}{p_2^*} = \frac{4}{3}\alpha.$$

At $\alpha = 7/9$, it is $(84, 81, 14)/179$, not $(28, 27, 6)/61$; the latter is not stationary for the declared channel. At $\alpha = 1$, state 3 receives no input from state 2 but can still decay into it: this is absence of incoming branching, not full dynamical decoupling of state 3. No general full-state convergence theorem for this three-state family is asserted here.

Finally, a channel alone fixes no energy cost. An explicit Stinespring isometry $V|\psi\rangle = \sum_j K_j|\psi\rangle \otimes |j\rangle_E$ shows that its implementation requires a supplied environment input and that discarded correlations reside in the environment. It does not fix Hamiltonians, switching work, bath renewal, clock cost or temperature. Those are additional entries in §11's resource account; no entropy-area or gravitational normalization follows from these fixed populations.

D.12 Event comparisons, composable histories and winding observability

D.12.1 A declared receipt alphabet does not specify its transition graph

The additional working notes [24] motivate the following conditional alphabet. A label contains incoming and outgoing Hamming weights $w, w' \in \{0, 1, 2, 3\}$ and parity $\epsilon \in \{0, 1\}$. Fixed-weight labels have $w' = w$ and no address, giving $|F| = 8$. One-step labels have $w' - w = +1$ or -1, three possible adjacent weight steps in each direction and address $a \in \{0, 1, 2\}$. Thus $|O| = |E| = 3 \cdot 3 \cdot 2 = 18$. These are coarse labels, not a count of underlying microstates or independently generated events. Their parity/address interpretation remains conditional on a supplied receipt map.

There are $\binom{44}{2} = 946$ unordered pairs of distinct labels. Pair comparison differs from history composition: the ordered history ℓ_1 then ℓ_2 requires at least $w_1' = w_2$. Applying that necessary weight condition yields:

Comparison block	All distinct unordered pairs	At least one weight-compatible ordering
F, F	28	4
O, O	153	72

Comparison block	All distinct unordered pairs	At least one weight-compatible ordering
E, E	153	72
F, O	144	72
F, E	144	72
O, E	324	108
Total	946	400

For F, F, weights must agree; only the two opposite-parity labels at each of four weights can form such a distinct pair. For O, O, the first weight step must end where the second starts: two successive step positions, each with 6·6 address/parity choices, give 72. Extinctions give the same count. Each onset or extinction label can compose with the two fixed labels at either endpoint, giving 72 in each mixed fixed block. For O, E, the two weight steps must be reverses: three step positions with $6 \cdot 6$ choices give 108. There are 520 ordered weight-compatible pairs when repetition is included. This last number is also $8^2 + 14^2 + 14^2 + 8^2$, from the incoming/outgoing label counts at each weight.

These are combinatorial compatibility results. Full bit patterns, address histories, transport gates, null events and frame consistency may impose further constraints. Neither the 946 comparisons nor all 400 weight-compatible pairs constitute a derived native transition graph. A partition by label type also need not be an invariant sector of an unspecified evolution operator.

The O, E block demonstrates a useful collision of equal counts. Its same-address subset has $3 \cdot 6^2 = 108$ pairs. Its reverse-weight-step subset also has $3 \cdot 6^2 = 108$ pairs, but the intersection contains only $3 \cdot 3 \cdot 2^2 = 36$. The address and weight-step predicates are independent restrictions in this declared product alphabet. Equality of their cardinalities does not identify the subsets, their admissibility or their dynamics.

D.12.2 When a receipt determines a winding or cost

Let $\mathcal{H}$ be a declared set of fine histories, $q : \mathcal{H} \to \mathcal{R}$ a receipt, and $W : \mathcal{H} \to \mathbb{Z}$ a winding increment. There exists a receipt-level function $\bar{W}$ on $q(\mathcal{H})$ with $W = \bar{W} \circ q$ if and only if W is constant on each fiber of q. Necessity follows by evaluating $\bar{W}$ on equal receipts. For sufficiency, define $\bar{W}(r)$ using any history in that fiber; the assumption makes the definition independent of the representative. The same criterion applies to any proposed cost functional.

A fixed-weight receipt alone does not satisfy this criterion for an unspecified winding. For a counterexample to the inference, extend the state to (w, ϵ, n), $n \in \mathbb{Z}$, and consider two candidate lifts that fix w, ϵ but update n to n or $n + 1$. Both project to the same fixed-weight receipt; their winding increments differ. This proves underdetermination from the label premises alone. It does not declare either lift admissible under a future complete UD source law.

Even when a winding increment ν is independently known, a quadratic burden $B(n) = \kappa n^2$ gives

$$\Delta B = \kappa(2n\nu + \nu^2).$$

The first and second positive unit steps cost κ and 3κ because their initial windings are 0 and 1. A cardinality ratio $324/108 = 3$ does not supply either winding, ν, κ, a probability law or an address-to-winding map. If $\nu = 0$ is separately established, then $\Delta B = 0$; a zero Hamming change alone does not establish that premise. A zero change of this burden stock also does not establish zero complete apparatus cost.

For example, the notes propose the cross-block cost $c(\ell_O, \ell_E) = 1 + 2\mathbf{1}_{a_O \neq a_E}$. This is an assigned pair cost, not a consequence of the pair counts. Replacing 2 by any nonnegative constant preserves the alphabet and all its counts. If the expression is asserted to be a metric on the whole label set, its nonzero diagonal first requires correction; the given formula is defined here only on the disjoint O, E block. Under an additional uniform distribution on those 324 pairs, the proposed mean is

$$\frac{108 \cdot 1 + 216 \cdot 3}{324} = \frac{7}{3},$$

which is a different object from the count ratio 3. Thus label counting, history eligibility, winding inference, signed burden change and complete resource cost require separate maps. The new exact replay enumerates the declared alphabet and checks these counts [24]; the fiber criterion and counterexample supply the general logical boundary.

D.13 A declared history graph and exact receipt requirements

D.13.1 Fine events, active receipts and nulls

This development model supplies an explicit graph for the one-pair alphabet of D.12; it is not a native admission law [25]. Let $\theta \in \{0,1\}^3$ and $g \in S_3$ act by permuting the three pair addresses. A primitive action has $\xi = 0$ or e_a, $a \in \{0,1,2\}$, and updates

$$\theta' = P_g \theta \oplus \xi.$$

All these actions are admitted in this reference graph. There are eight vertices and $8 \cdot 6 \cdot 4 = 192$ action-distinct directed edges, including loops and parallel edges. This count is not an identification with the different $S_4 \times \Theta$ product counted in §2.2. Repeated action choices define the fine-history space. Eligibility gates, geometry changes, control sources and resource exhaustion are not supplied by this declaration.

For an edge other than $(g, \xi) = (1, 0)$, record

$$\rho(e) = (w, w', a, \epsilon), \quad w = |\theta|, \quad w' = |\theta'|, \quad \epsilon = \mathrm{parity}(g),$$

using $a = \emptyset$ when $\xi = 0$. Record the identity/no-flip action as a distinct null symbol $\perp$. This gives 44 active receipt labels plus one null, not 44 total labels. The active receipt fibers have sizes 2, 3, 6 and 9 with multiplicities 2, 26, 14 and 2; the null fiber has eight edges. Counts refer to varying fine initial states as well as controls. They are not probabilities without a measure.

For any fixed θ of incoming weight w, every active receipt with that weight can be realized by some admitted action. For weights 1 and 2, the three even permutations and the three odd permutations each move the exceptional bit to every address. Weights 0 and 3 are fixed by all permutations. Thus every weight-compatible active receipt word lifts to a path from every initial configuration of its first weight in this all-open graph. The 400-pair count in D.12 is consequently attained at this coarse level here; more restrictive native gates could reduce it. A null retains the state and does not remove the need for weight compatibility across it.

D.13.2 A normalized kernel family does not select a preference

For each weight choose probabilities $b_w, d_w \geq 0$ with $b_w + d_w \leq 1$, $b_3 = d_0 = 0$, and put $h_w = 1 - b_w - d_w$. Draw g uniformly from S_3. With $y = P_g \theta$, define the complete action kernel

$$K(g,\xi \mid \theta) = \frac{1}{6}\begin{cases} h_w, & \xi = 0, \\ b_w/(3-w), & \xi = e_a,\ y_a = 0, \\ d_w/w, & \xi = e_a,\ y_a = 1. \end{cases}$$

Only existing zero or one bits enter the respective branches; the boundary cases have zero total probability and no division by zero. Summing all actions gives one. Each initial state of weight w has the same next-weight law

$$Q_{w,w+1} = b_w, \quad Q_{w,w-1} = d_w, \quad Q_{w,w} = h_w.$$

The null probability is $h_w/6$. It differs from the no-flip probability h_w and from the probability that the Boolean endpoint is unchanged: $h_w/\binom{3}{w}$. Frame actions can occur without changing that endpoint. One model tick is supplied; no physical waiting-time distribution or clock calibration is derived.

For the reference choice $h_w = 1/4$, $b_w = (3-w)/4$ and $d_w = w/4$, all 24 controls have probability $1/24$. The coarse matrix and stationary row vector are

$$Q = \frac{1}{4}\begin{pmatrix} 1 & 3 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix}, \qquad \pi = \frac{1}{8}(1,3,3,1),$$

with eigenvalues $1, 1/2, 0, -1/2$. This is a prepared classical stochastic model, distinct from the coherent skew flow.

There is no symmetry-selected stationary preference in this family. Given any strictly positive probability vector $\tilde{\pi}$, choose positive adjacent fluxes J_0, J_1, J_2 small enough that $J_{w-1} + J_w < \tilde{\pi}_w$, with $J_{-1} = J_3 = 0$, and set

$$b_w = J_w/\tilde{\pi}_w, \qquad d_w = J_{w-1}/\tilde{\pi}_w.$$

These probabilities give detailed balance $\tilde{\pi}_w b_w = \tilde{\pi}_{w+1} d_{w+1}$ and positive holding probabilities. The fine stationary distribution is $\tilde{\pi}_{|\theta|}/\binom{3}{|\theta|}$, since the uniform frame draw makes the endpoint uniform within each outgoing weight orbit. Thus the same all-open graph and permutation symmetry admit every positive coarse stationary law. The construction proves nonselection; it does not recommend these freely chosen rates as native UD dynamics.

D.13.3 Scalar frame consistency excludes loop winding

An additive integer or real edge functional is additional data. A useful restricted class can nevertheless be characterized. Let $f(\theta, g)$ be its increment for a frame move and $t(\theta, a)$ its increment for a single flip. Assume:

1. Frame composition obeys $f(\theta, hg) = f(\theta, g) + f(P_g\theta, h)$.
2. Both increments are scalars invariant under simultaneous relabeling: $f(P_k\theta, kgk^{-1}) = f(\theta, g)$ and $t(P_k\theta, k(a)) = t(\theta, a)$ for every $k \in S_3$.
3. Flip reversal changes the sign: $t(\theta \oplus e_a, a) = -t(\theta, a)$.
4. A composite frame-then-flip action has increment $f(\theta, g) + t(P_g\theta, a)$; no extra composite increment is introduced.

Then there is a function F of Hamming weight such that every history has total increment

$$W(\gamma) = F(w_{\text{final}}) - F(w_{\text{initial}}).$$

In particular every closed history has zero increment. These hypotheses define the class being excluded from nontrivial scalar loop winding; they are not established premises for every possible UD winding law.

To prove the frame statement, define $m(\theta) = \frac{1}{6}\sum_g f(\theta, g)$. The composition identity gives $m(P_h\theta) = m(\theta) - f(\theta, h)$, while scalar relabeling invariance gives $m(P_h\theta) = m(\theta)$. Hence $f = 0$. Upward flip edges have three relabeling orbits, indexed by incoming weights 0, 1 and 2. Call their increments a_0, a_1, a_2; reversal fixes the downward increments. Set $F(0) = 0$ and $F(w) = \sum_{j<w} a_j$. Each elementary increment is the difference of this potential, and the composite rule preserves it. Summing telescopes. There are exactly three free parameters, with integer parameters for integer increments. The finite certificate independently verifies rank 69 for 462 constraints on 72 frame/flip values, leaving precisely this three-dimensional space [25].

D.13.4 A graph-cycle diagnostic and its finite record size

To test receipt sufficiency outside that restricted class, declare a different edge functional. At $\theta = 000$, assign $+1$ to the no-flip frame action $c = (012)$ and -1 to c^{-1}; assign zero to other edges. This functional is antisymmetric under edge reversal, but deliberately fails the frame-composition identity and full S_3 scalar covariance. It is a chosen graph-cycle counter, not a derived physical winding. Treating different frame words as the same group action would invalidate this choice.

Both nonzero actions begin and end at 000 and emit the same active receipt $r = (0, 0, \emptyset, 0)$. For the word r^N, the possible totals are

$$W_N \in \{-N, -N+2, \dots, N\}.$$

All $N + 1$ values occur. Even when the initial and final Boolean states, full receipt word and length N are known, exact recovery of total W_N therefore requires a supplement with at least $N + 1$ distinct values. A fixed-length binary supplement needs

$$B_{\min} = \lceil \log_2(N + 1) \rceil$$

bits; emitting W_N attains the cardinality bound. This is aggregate recovery, not recovery of the full frame history. Under the reference kernel, conditional on this word, the two actions are equiprobable independently, giving binomial probabilities for these totals. The word itself has probability $(1/12)^N$ from initial 000. The support bound is a worst-case exact requirement, not an assertion about average cost.

If a receiver, supplied with the known incoming weight, merges the null action with r, the same code word instead hides increments $-1, 0, +1$. It then permits every total from $-N$ to N, requiring $2N + 1$ supplemental values and $\lceil \log_2(2N + 1) \rceil$ fixed-length bits. This is a separate 44-code observation that conflates nulls with active events. It is not the 44-active-plus-null contract above. The unflagged word can be reconstructed from the flagged word and initial weight by carrying the weight across nulls; the two standalone symbol maps are not globally ordered because $\perp$ alone omits that weight. Exact winding over unbounded horizons cannot fit into a fixed finite supplemental alphabet under either observation.

For a supplied burden $B(n) = \kappa_B n^2$, known initial counter n_0 and recovered total W_N suffice to compute

$$\Delta B = \kappa_B(2n_0 W_N + W_N^2).$$

This is a sufficient receipt for the signed stock change, not necessarily a minimal one for that change alone; different windings can give the same value. The counter is distinct from the orientation sign called $n_{\text{seam}\to\text{chain}}$ in the source ledger. Neither κ_B, physical winding, readout acquisition, work, renewal cost nor a boundary-area density is derived. The capacity bounds are conditional on the receipt word and its known length; those side-information resources cannot be omitted from an apparatus ledger. The result identifies what a specified history functional needs recorded before a resource comparison can be completed.

D.14 Composition of controls and the limits of phase extensions

D.14.1 Primitive controls generate an affine action group

The 24 primitive controls of D.13.1 are not closed under composition. Their closure is

$$G_{\text{aff}} = \mathbb{F}_2^3 \rtimes S_3, \qquad (v, g)(u, h) = (v \oplus P_g u, gh), \qquad |G_{\text{aff}}| = 48.$$

Indeed, the primitive set contains all coordinate permutations and each single-bit translation. Composing the latter generates every translation in $\mathbb{F}_2^3$; conversely, affine composition never leaves the displayed set. Its action on the eight configurations is faithful: evaluation at zero recovers v, and evaluation at the three unit vectors recovers P_g. This identifies a consequence of repeated controls in the declared graph, not a new native admission or rate law [26].

There is also an explicit abstract isomorphism $G_{\text{aff}} \cong C_2 \times S_4$. Center the Boolean cube: an affine action becomes a signed permutation matrix $M = \text{diag}((-1)^{v_j})P_g$. Its determinant sign gives $\delta \in C_2$, and its permutation of the four opposite-vertex pairs gives $\sigma \in S_4$. A signed permutation fixing all four pairs is $\pm I$. To see this, the four diagonal lines are in general position: three span the space and the fourth has all three coordinates nonzero in that basis, forcing a common eigenvalue. The determinant distinguishes I and $-I$. Hence (δ, σ) is injective and, since both groups have 48 elements, bijective.

An endpoint transformation does not identify the intervening word. In particular, twice applying a single flip acts as the identity while using two primitive steps. Neither the 48-element closure nor a receipt of the final transformation determines a history cost or the graph-cycle counter of D.13.4. Thus the state-action group, event word and retained history functional remain separate objects.

D.14.2 A finite phase label does not establish physical winding

Appending a central C_{13} factor produces a mathematically admissible group of order 624 with quotient S_3 after removing $\mathbb{F}_2^3 \times C_{13}$. This does not identify its 104-element kernel with the four-element Klein kernel of $S_4 \to S_3$. No native connection, parallel transport or integer winding follows from the appended label. A C_{13} character can distinguish a declared counter only modulo 13; an unbounded integer counter still requires a lift and its acquisition contract.

Nor does the affine group force projective phases to be trivial. With the preceding coordinates let $p(g)$ be the parity of $\sigma(g)$. For the Pauli matrices X, Z, define

$$R(g) = X^{\delta(g)} Z^{p(g)}, \qquad R(g)R(h) = \alpha(g, h)R(gh), \qquad \alpha(g, h) = (-1)^{p(g)\delta(h)}.$$

The additive characters p, δ make α a normalized two-cocycle. Choose a central element with $(\delta, \sigma) = (1, 1)$ and an element with $\delta = 0$ and odd σ. They commute in the group, but their lifts

X, Z anticommute. Rephasing either lift cannot change this commutator, so the projective class is nontrivial. Letting C_{13} act trivially preserves this counterexample for the extended group. This disproves the imported assertion that all projective classes vanish; it does not assert a physical anomaly. A physical gauge-anomaly calculation requires a specified physical action and field content, neither supplied here. The exact replay checks the affine isomorphism, all 48^3 cocycle triples and this witness [26].

D.15 A carrier-driven admission candidate with finite activity

D.15.1 Declare the response rule rather than infer it from a stock identity

Keep a single filled tetrahedron, the signed S_4 action and the constant generator $A(a, b, c)$ of §5.1. Write its real state as $x(s)$, with $\|x\| = R > 0$. For each opposite-edge pair p, let D_p project onto its two edge coordinates and define

$$q_p = x^T D_p x, \qquad 0 \le q_p \le R^2.$$

These are squared carrier amplitudes, not physical energies. Introduce three response-memory bits ϑ_p and two supplied common thresholds $0 < \ell < h < R^2$. The candidate rule sets $\vartheta_p : 0 \to 1$ on reaching $q_p \ge h$, and $\vartheta_p : 1 \to 0$ on reaching $q_p \le \ell$. Between the thresholds it retains its previous value. Start with a supplied consistent memory: a zero bit has $q_p < h$ and a one bit has $q_p > \ell$. At each first guard hit update every enabled bit in one batch. Leave x unchanged and continue the same A.

This is a deterministic passive hybrid readout once its initial state and thresholds are supplied [27]. The common thresholds make it covariant under fixed vertex relabeling, since signed permutations transport D_p to the corresponding pair projector and commute with A. All integrity gates are held open; forced resets, changing gates or feedback to A are outside this contract. The response memory may be used as a candidate binary label, but no identification with native closure, obligation saturation or geometric replacement is derived. Hysteresis and dwell-time analysis are established control methods [28]; the following constants and collision example concern this particular carrier.

D.15.2 Exact stock-speed and event-count bounds

For any orthogonal projector D and skew A, set $B = DA(I - D)$ and $\beta_D = \|B\|$. Direct differentiation gives

$$q' = x^T(DA - AD)x = 2(Dx)^T A(I - D)x, \qquad |q'| \le 2\beta_D\sqrt{q(R^2 - q)} \le \beta_D R^2.$$

The commutator $DA - AD$ is symmetric and, in $D \oplus (I - D)$ blocks, has off-diagonal blocks B, B^T; its norm is β_D. For an opposite-edge pair of the tetrahedron, the two nonzero rows of B have Gram matrix

$$BB^T\big|_{\operatorname{im} D_p} = 2(a^2 + b^2)I_2, \qquad \beta = \sqrt{2(a^2 + b^2)}.$$

Each edge has two vertex and two face incidences; opposite edges have disjoint vertex and face supports. This proves the identity and explains why the cell/face coefficient c does not enter the direct bound. If $a = b = 0$, the pair stocks are constant and no subsequent switches occur from a consistent initialization.

For $\beta > 0$, consecutive switches of any one address are separated by at least

$$d_* = \frac{\arcsin\sqrt{h/R^2} - \arcsin\sqrt{\ell/R^2}}{\beta} \;\geq\; \frac{h-\ell}{\beta R^2} > 0.$$

To prove this, a switch back requires traversal of the threshold band. On that band, differentiating $\arcsin\sqrt{q/R^2}$ bounds its absolute speed by β. Integrating along the traversal proves the first inequality, and $2\sqrt{q(R^2-q)} \leq R^2$ proves the second. Nonmonotone motion cannot shorten this required traversal. For the canonical generator, $\beta = 2$, the angular bound is attainable: if e is a unit coordinate vector on one edge in the pair, the invariant plane spanned by $e, Ae/2$ has a unit-norm trajectory with $q_p = \sin^2(2s)$. Its increasing crossing time between the two levels equals d_*.

Consequently, in an interval of length S, the total number of individual toggles is bounded by

$$N_{\text{toggle}}(S) \leq 3\,(1 + \lfloor S/d_* \rfloor)\,.$$

The batch count is no larger. Thus no finite interval contains infinitely many switches. This is a per-address dwell bound and a finite-activity result, not a uniform positive separation between all distinct event times. Different addresses can cross arbitrarily close together.

D.15.3 Bounded observation error and crossing-time sensitivity

If the observed stocks satisfy $|\hat{q}_p - q_p| \leq \varepsilon < (h-\ell)/2$, consecutive switches of an address require the true stock to move between at least $h-\varepsilon$ and $\ell+\varepsilon$. Replace d_* above by

$$d_{*,\varepsilon} = \frac{\arcsin\sqrt{(h-\varepsilon)/R^2} - \arcsin\sqrt{(\ell+\varepsilon)/R^2}}{\beta} \;\geq\; \frac{h-\ell-2\varepsilon}{\beta R^2} > 0.$$

The event-count bound follows with this replacement. The strict half-gap condition is necessary for a uniform count bound based only on this error amplitude and carrier constants. For the canonical flow choose orthonormal opposite-edge vectors e_1, e_2 in the same pair and put $w_i = Ae_i/2$. The unit trajectory

$$x(s) = \frac{\cos(2s)e_1 + \sin(2s)w_1 + \sin(2s)e_2 - \cos(2s)w_2}{\sqrt{2}}$$

has constant $q_p = 1/2$. With $\ell = 3/8$, $h = 5/8$ and smooth measured stock $\hat{q}_p = 1/2 + \sin(Ms)/8$, the error is exactly half the gap and the controller switches at successive maxima and minima. Arbitrarily large M gives arbitrarily many toggles on a fixed interval while the true stock remains constant. This refutes a uniform amplitude-only count bound at the boundary; it does not assert infinite activity for any one fixed smooth noise signal.

For a well-defined continuous measured guard signal the first-hit rule applies directly; at supplied sample times the same count bound holds, but sampling delay needs a separate bound. If $\hat{x} = x + e$, $\|e\| \leq \delta$, then

$$|\hat{x}^T D_p \hat{x} - x^T D_p x| \leq 2R\delta + \delta^2.$$

Hence $\delta < \sqrt{R^2 + (h-\ell)/2} - R$ is sufficient for a positive noise-adjusted gap. This is a declared norm-error contract, not an inferred apparatus noise law.

Event timing needs more than a gap. Suppose a matched ideal rising crossing s_* is isolated, the previous response state agrees, no earlier noisy guard hit is possible, and $q'_p \geq \nu > 0$ throughout the local band $[h-\varepsilon, h+\varepsilon]$. Continuity of the measured signal then gives

$$|\widehat{s}_* - s_*| \leq \varepsilon/\nu.$$

Indeed the measured first hit lies between the true crossings of $h-\varepsilon$ and $h+\varepsilon$, each at most ε/ν from s_*. Descending crossings have the corresponding $-q'_p$ bound. Without isolation and a lower slope, small errors can create, erase or reorder crossings; no such timing guarantee is claimed at grazing or simultaneous events.

D.15.4 Symmetric collisions obstruct a universal one-pair selector

For the canonical generator, start from the unit vertex state $x(0) = e_0$ and $\vartheta = 000$. All three pair stocks are

$$q_p(s) = \tfrac{1}{4}\sin^2(2s).$$

Choose, solely as an exact test fixture, $\ell = 1/16$ and $h = 3/16$. All three bits turn on at $s = \pi/6$ and off at $s = 5\pi/12$, with crossing slopes $\pm\sqrt{3}/4$. The stabilizer of vertex 0 is S_3 and permutes the three pair addresses transitively while fixing the complete carrier trajectory and initial response. A deterministic equivariant rule required to select exactly one of these enabled addresses would have to return an address fixed by that stabilizer. None exists. It can hold or select the invariant batch; a one-address choice requires an additional reference/asymmetric state or a stochastic rule. Symmetry requires equal probabilities among the three if that is the only available information, but does not source randomness or its clock.

The batch $000 \to 111$ is not a primitive edge of D.13's no-flip/one-flip graph. Its endpoint transformation belongs to the 48-element closure in D.14. Serializing it into three one-flip events creates six possible orders; if a realized order matters, recovering it requires six values, hence three worst-case fixed-length bits conditional on the known batch. These are information requirements, not a derived physical work cost or three physical elapsed ticks.

There is also no uniform global dwell bound even away from exact collision. For $x(0) = (e_0 + \eta e_1)/\sqrt{1+\eta^2}$ with small $\eta > 0$,

$$q_A = \frac{(1-\eta)^2}{4(1+\eta^2)}\sin^2(2s), \qquad q_B = q_C = \tfrac{1}{4}\sin^2(2s).$$

The A onset occurs just after the B/C onset, with a separation tending to zero as $\eta \to 0$. Per-address dwell and finite activity remain valid. Constructor checks cover the exact carrier, all signed symmetries, stock identities, transverse fixture and collision stabilizer [27].

The result supplies a testable candidate readout rule with explicit assumptions and proves its event and noise limits. Its thresholds, memory preparation, continuous observation, collision semantics and implementation resources remain supplied. It does not select native events from the carrier alone, alter the reconciled obligation/flux audit, or close G05/G06, physical winding, apparatus renewal or entropy-area normalization.

D.16 Sampled acquisition and receipts of uncertain event times

D.16.1 A dwell bound does not guarantee threshold detection

Keep D.15's passive fixed-carrier model. Its per-address dwell bound concerns the ideal response memory after a guard hit. It does not require the stock to remain above the upper threshold until a sampler reads it. For example, on the canonical vertex trajectory $q_p = \frac{1}{4}\sin^2(2s)$ choose $h = 1/4$ and $\ell = 1/8$. The ideal inclusive guard turns on at $\pi/4$ and off at $3\pi/8$. For every odd integer N, the grid $s_k = k\pi/(2N)$ misses every maximum: equality would require $2k = (2j+1)N$, impossible by parity. Direct threshold polling therefore remains off even as the grid becomes arbitrarily fine.

This is a limitation of the threshold-polling observer, not an impossibility theorem for model-assisted reconstruction from richer data. Transverse high excursions can also be arbitrarily narrow near grazing; a uniform completeness guarantee needs a quantitative excursion condition rather than a cadence alone. The finite-activity theorem in D.15 remains valid.

D.16.2 Certified guards and sufficient conditions for faithful acquisition

Let actual sampling times have maximum gap $\Delta > 0$ and let $|y_{p,k} - q_p(s_k)| \le \varepsilon$. Initialize the sampled response equal to the ideal one. Use the conservative guards

$$0 \to 1 \quad \text{if } y_{p,k} - \varepsilon \ge h, \qquad 1 \to 0 \quad \text{if } y_{p,k} + \varepsilon \le \ell;$$

otherwise retain the sampled bit. Update all enabled bits in a detection batch, leaving the carrier unchanged. A confirmation implies that the true stock is on the appropriate side of its ideal threshold. Large error can prevent confirmation altogether. Unlike D.15's raw noisy threshold rule, these interval guards need no half-gap condition merely to exclude a wrong-side confirmation; completeness has the additional conditions below [29].

For each ideal rising event at τ, assume the true stock reaches $h + 2\varepsilon$ at some $u \le \tau + 2\varepsilon/\nu$, and stays at least that high on $[u, u + \Delta]$. A sufficient reach condition is $q_p' \ge \nu > 0$ until reaching that level. For a falling event use $\ell - 2\varepsilon$ and $-q_p' \ge \nu$. These are supplied event-wise excursion and persistence hypotheses; they are not consequences of skewness alone.

Then the matched sampled event is confirmed with delay

$$0 \le s_{\mathrm{det}} - \tau \le D, \qquad D = 2\varepsilon/\nu + \Delta.$$

Proof. A sample occurs in $[u, u + \Delta]$. For a rise, its value satisfies $y \ge q - \varepsilon \ge h + \varepsilon$, so the rising guard fires if it has not already done so. A matched confirmation cannot precede the ideal event, because its true stock must already have reached the ideal threshold. The falling proof is the same with inequalities reversed.

If these hypotheses hold for every event and $D < d_*$, where d_* is D.15's ideal per-address dwell bound, induction gives exactly the same toggle word at each address: every event is confirmed before the next opposite event, and no extra matched event can occur between ideal switches. This claim preserves address-wise words, not necessarily the global order or simultaneity of different addresses. For a finite acquisition window the guarantee covers events at least D before its end; later events require a tail acquisition or an unresolved end-window receipt.

D.16.3 Time intervals preserve what detection order cannot

With exact detection time s_i, retain an event-time interval $I_i = [s_i - D_i, s_i]$. With timestamp error at most j, use

$$I_i = [\hat{s}_i - D_i - j, \hat{s}_i + j].$$

Declare event i definitely earlier than k only if $\sup I_i < \inf I_k$. Overlapping intervals do not certify order or simultaneity. A sufficient global ordering condition for ideal times $\tau_i < \tau_k$ is $\tau_k - \tau_i > D_i + 2j$, since delays are one-sided. D.15 supplies no uniform gap between different addresses, so no positive fixed cadence guarantees global ordering on that full domain. The bound Δ refers to actual gaps; if nominal periodic samples have time jitter j_s, use at most $\Delta_{\text{nominal}} + 2j_s$ instead.

Even an ideal simultaneous batch may split into several detection batches under address-wise error. Such a log records acquisition order. It must not be used as a fine carrier-event order without further evidence. If $m!$ different serial orders remain possible and the desired functional distinguishes all of them, recovering a realized order requires at least $\lceil \log_2(m!) \rceil$ additional fixed-length bits, conditional on the known event set. This is a necessary storage condition, not a mechanism for observing the order; guessing those bits adds no evidence. True batches and unresolved order remain valid receipt objects.

D.16.4 Quantization, an exact fixture and a finite storage ledger

A b-bit uniform quantizer on $[0, R^2]$ with bin-midpoint outputs has maximum quantization error $R^2/2^{b+1}$. Clipping an analog reading to that interval cannot increase its error relative to a true stock in the interval. With analog error at most ε_a, use

$$\varepsilon = \varepsilon_a + R^2/2^{b+1}.$$

A sufficient design inequality for the delay certificate is

$$2\varepsilon_a + R^2/2^b < \nu(d_* - \Delta), \qquad \Delta < d_*.$$

The excursion hypotheses must still be checked. These are logical precision and cadence budgets, not an ADC implementation or a physical energy law.

For the canonical unit vertex trajectory, use $R = 1$, $\ell = 1/16$, $h = 3/16$, $b = 6$, $\varepsilon_a = 0$ and $\Delta = 1/64$. Then $\varepsilon = 1/128$. On the required rising/falling bands, the speed is at least $\sqrt{39}/16 > 3/8$; choose $\nu = 3/8$. The confirmation core has length at least $3/64 > \Delta$: its stock margin to the peak or trough is $3/64$ and $|q_p'| \le 2$. The sinusoidal upper and lower cores are each a single interval. Thus

$$D = \frac{2(1/128)}{3/8} + \frac{1}{64} = \frac{11}{192} < \frac{1}{16} \le d_*.$$

Six bits are the minimum for the chosen conservative test $D < 1/16$; five give $19/192 > 1/16$. This uses the linear dwell certificate rather than optimizing the angular one, and is not a globally optimal digitizer lower bound.

On $s_n = n/64$, $0 \le n \le 128$, the deterministic midpoint-quantized observer records masks 111 at ticks 34 and 84. Exact rational sine enclosures certify every bin and guard. Allowing arbitrary stock errors within $1/128$ gives per-address onset ticks 34–36 and extinction ticks 84–87, with all twelve combinations permitted by the replay envelope. All are within the proved delay bound. The $12^3 = 1728$ joint combinations can produce up to six detection batches from the two ideal batches; no probability or independence law is assumed.

There are 129 scans and 387 scalar stock values. The declared current buffers use 18 stock bits, three response bits, an eight-bit tick counter and a three-bit mask buffer. Retaining every quantized stock value separately uses 2322 bits. A dense response-word format uses $3(129) = 387$ bits including the initial response. A sparse format with a three-bit initial response, an eight-bit count, and an eight-bit tick plus three-bit mask per record uses 33 bits for the deterministic two-record trace, or at most 77 for the six-record error envelope. These formats encode the observed response log; they do not replace the raw stock archive when that is required. Interval interpretation, scale, thresholds, quantizer, clock and error contracts require separately supplied metadata.

Storage capacity is not automatically information gained; a completely known deterministic preparation may make its trace predictable. Nor is the log the whole apparatus: sensor preparation, back-action, calibration, reference/clock supply, reliable retention, coding/processing and renewal remain uncosted physical resources. This package closes a conditional finite sampling and receipt contract. Native event selection, physical implementation, complete energy cost and entropy per boundary area remain open.

D.17 Receipt consistency, covariance and errors it cannot reveal

D.17.1 The declared signless tree map

The filled tetrahedron has a Hasse graph with 15 simplex vertices and 28 adjacent-degree containment edges. Choose the 14-edge spanning tree and 14 chords specified in the certificate [30]. For a declared latent field $z \in \mathbb{R}^{15}$, let $S, H \in \{0,1\}^{14\times 15}$ map z to endpoint sums on tree and chord edges. These are signless observation matrices, not the signed boundary operator or skew generator.

The graph is bipartite by simplex-degree parity. Let v have value -1 on even degrees and $+1$ on odd degrees. Then $Sv = Hv = 0$. Connectivity gives $\ker S = \operatorname{span}\{v\}$, so every chord row is in the tree row space. Along its odd-length tree path, alternating the endpoint sums cancels internal vertices and retains the two endpoints. Hence

$$H = CS, \qquad C \in \{-1, 0, 1\}^{14\times 14}.$$

For the supplied tree, $\operatorname{rank} S = 14$ and $\operatorname{rank} H = \operatorname{rank} C = 10$. The longest chord path has seven edges. A different tree can change C and its error amplification. The graph argument is standard incidence/path algebra; the certificate identifies this particular split, not a new general reconstruction method.

A possible magnitude-receipt application requires a separate map. On full nonzero support, if a product receipt is $|J_{ij}| = 2|B_{ij}||c_i||c_j|$, define $z_i = \log|c_i|$ and $y_{ij} = \log(|J_{ij}|/(2|B_{ij}|))$. Then $y_{ij} = z_i + z_j$. Zero amplitudes, phase/sign information and calibration require additional treatment. Applying these signless matrices directly to the chain amplitudes instead defines another observable. The null direction v is a measurement gauge: $Av \neq 0$ for the native canonical generator, so it is not a harmonic carrier mode.

D.17.2 What zero residual covariance establishes

For noiseless observations $Y_S = Sz$ and $Y_H = Hz$,

$$r = Y_H - CY_S = 0$$

holds for every realization, independently of Gaussianity or the dynamics that generated z. With finite second moments, $\Sigma_{HH} = C\Sigma_{SS}C^T$ and $\Sigma_{HS} = C\Sigma_{SS}$. If $M = (I; C)$, any positive semidefinite supplied training covariance produces

$$\Sigma_{28} = M\Sigma_{SS}M^T \succeq 0, \quad \operatorname{rank}\Sigma_{28} = \operatorname{rank}\Sigma_{SS}.$$

The equality of ranks follows because M has the left inverse selecting its first 14 rows. The 14 independent columns of $(-C^T; I)$ lie in the covariance kernel. They span the entire kernel only when Σ_{SS} is positive definite; otherwise the kernel is larger.

Thus zero residual from the same generated latent field confirms the enforced observation identity. It does not independently validate the carrier evolution, a source law or a physical prediction. Any alternative latent history passed through the same matrices satisfies it too. A covariance propagation identity is not an empirical held-out success.

D.17.3 A residual error contract and its blind subspace

Now declare independently acquired channels with errors, allowing arbitrary correlations:

$$\widetilde{Y}_S = Sz + e_S, \qquad \widetilde{Y}_H = Hz + e_H, \qquad r = e_H - Ce_S.$$

Writing $R_{ij} = \operatorname{Cov}(e_i, e_j)$, finite second moments give

$$\operatorname{Cov}(r) = R_{HH} + CR_{SS}C^T - R_{HS}C^T - CR_{SH}.$$

The cancellation of z is pointwise, so this formula does not require the errors to be independent of z. For zero-mean independent equal-variance errors across all 28 channels it becomes $\sigma^2(I + CC^T)$, positive definite when $\sigma > 0$. For componentwise error bounds,

$$\|r\|_\infty \le \varepsilon_H + 7\varepsilon_S$$

on this tree. The row-specific coefficient is its path length, ranging from three to seven here. These are supplied noise contracts, not a derived sensor law. The residual mean must also be tested: a constant nonzero $e_H = b$, $e_S = 0$ has zero covariance but residual b.

Finally set $L = (-C, I)$. Its full blind error subspace is

$$\ker L = \{(u, Cu) : u \in \mathbb{R}^{14}\}, \qquad \dim\ker L = 14.$$

Such errors change both blocks consistently and produce zero residual. Since S is surjective, each is the observation of some latent-field perturbation. Additional roots or stock references can resolve aspects of the measurement gauge, but a dynamics test still needs a specified temporal prediction or other independent observable. The residual can detect failures of the declared receipt consistency relation; passing it cannot establish that all state estimates, event sources or physics are correct. This distinction qualifies how the sampled receipts of D.16 can be audited without claiming unearned predictive validation.

Appendix E. Data, code and development declarations

The preserved `UD_CONDITIONING_REVIEW_v0.8_PACKAGE.zip` contains the prior manuscript, the conditioning report, exact Gram/LDL/Sturm certificates, current replay and the preserved version-0.7 gluing package. Run `verify_conditioning.py` with SymPy, NumPy and SciPy; its required source certificates are in `baseline/`. It regenerates the conditioning certificates and `CONDITIONING_CHECKS.json`. Prior source and replay scopes remain unchanged. `ENVIRONMENT.json` records the constructor environment.

The preserved `UD_PARTNER_REVIEW_v0.9_PACKAGE.zip` adds the prior revised manuscript, the partner-work audit, the original partner DOCX and its paragraph/Office Math extracts, an explicit claim-disposition register, `verify_partner_findings.py`, `PARTNER_CHECKS.json` and the positive-metric certificate. Run the new replay with SymPy; it executes no imported partner code. The archive preserves the full version-0.8 conditioning package. The imported working notes are unreviewed claim data; only independently rederived material is integrated. Their unattested physical, modular-filter and holonomy interpretations are not promoted. Approximate recurrence and metric-class qualifications also have written proofs in D.10.

The preserved version-0.10 additional-research package contains this manuscript, its companion audit, the independently authored `verify_additional_research.py`, exact check output, source register, claim dispositions and the five new input documents. Original DOCX and governance/ledger bytes are preserved inside the source archive with their hashes. The version-0.9 manuscript is preserved as a baseline; its complete earlier evidence package remains pinned by the source register and prior capture commit. The new replay checks event counts/composability, selected auxiliary finite-group facts and counterexamples to imported transport claims. It does not repeat every earlier replay. Auxiliary group/transport calculations are retained in the companion audit rather than admitted as UD physical correspondences. All additions remain development, with no registry promotion or independent review.

The preserved version-0.11 history-receipt package contains this manuscript, the companion finite-history report, `verify_history_receipts.py`, exact receipt/kernel checks and the full integer scalar-increment constraint certificate. It preserves the version-0.10 manuscript and pins its earlier evidence release in the source register. The new replay concerns the declared all-open Boolean/permutation graph and its two receipt contracts; it is not a replay or promotion of all historical UD claims. Null probabilities, scalar covariance, frame composition, supplied model ticks and the graph-cycle diagnostic's failed consistency hypotheses are explicit.

The preserved version-0.12 additional-research package adds three original partner DOCX files, paragraph and Office Math extracts, an audit with explicit claim dispositions, and the independently authored `verify_research_batch3.py`. Its 19 exact check families cover affine composition, a projective cocycle counterexample, the corrected order-104 finite-field block, receipt partitions and conditional scattering/noise benchmarks. D.14 integrates the affine result and the phase-extension limits. Neither the reference scattering networks nor an optional central phase factor is a derived native physical correspondence. The full eight-terminal floating-probe noise remains open. The version-0.11 manuscript is preserved and its evidence package pinned by hash and capture commit.

The preserved version-0.13 admission package adds a passive carrier-driven hysteresis candidate, its exact stock-speed, per-address dwell, bounded-noise and isolated-crossing bounds, a sharp half-gap noise counterexample, and a symmetry collision obstruction. The replay `verify_admission.py` independently constructs the incidence matrices and all signed actions. Twenty check families pass, including an explicitly numerical timing corollary; the general estimates have written proofs in D.15. The candidate holds integrity gates open and leaves the carrier unchanged, with thresholds, response memory and observation supplied. It does not

replace a native admission law or any frozen obligation/flux protocol. The package preserves the v0.12 manuscript and pins its evidence release and GitHub capture.

The preserved version-0.14 sampled-acquisition package adds a polling counterexample, certified interval guards, conditional completeness/delay proofs, uncertain-time receipts, a digitizer precision/cadence budget and explicit acquisition/storage counts. The new replay `verify_sampled_acquisition.py` passes 18 check families. It constructs exact rational enclosures for all guards in a finite synthetic trace and propagates a joint bounded-error envelope; no noise probabilities or physical measurements are supplied. The v0.13 manuscript and selected source bytes are preserved, with its complete evidence package pinned by SHA-256 and capture commit. This closes a conditional passive observation contract, not a physical sensor, native source law or complete apparatus-energy comparison.

The current version-0.15 additional-research package preserves four supplied DOCX originals, complete main-document XML, paragraph/table extracts and Office Math equations. Main-document hashes identify one repeat of S9 and one duplicate pair within this batch. The new independent replay `verify_research_batch4.py` passes 25 check families: signless covariance/path identities, noise and blind-error qualifications, Boolean orbit counts, and auxiliary cyclic-cocycle/modular repairs. D.17 integrates the receipt/error findings. The companion audit records exact contradictions to imported modular and physical-protection claims, with precise primary references; optional topological models are not identified with native UD physics. Original source code is not executed, and prior evidence is pinned rather than all replayed.

Source recovery is pinned to repository base commit `4b9c1f7ba03f76324b792f389517161d2bc47e05` and first-package commit `f274108856caecd2504f28a6729a91ce93846893`; the source register records the inspected bytes. These identifiers establish provenance rather than review acceptance. Package verification does not replay every proof or apparatus theorem in Papers I-IV; inherited results must be reviewed in those sources.

ChatGPT/Codex assisted with synthesis, source checking, symbolic/numerical calculations and preparation of this review candidate. This is a development disclosure. The named author must verify the final text, accept responsibility for it, and complete the target venue's authorship, funding, competing-interest and assistance declarations before submission. No declaration of funding or competing interests is inferred from missing source information.

References

1. Mayes, Benjamin Walker. *Finite Tetrahedral Complexes, Skew Support Flow and Contact Readiness*. Paper I, consolidated review draft 1.1.1, 4 October 2026. M5 release, `source/paper-1.md`.

2. Mayes, Benjamin Walker. *Projective Reconstruction, Complex Phase Receipts and Frozen-Tree Validation*. Paper II, consolidated review draft 1.1.1, 4 October 2026. M5 release, `source/paper-2.md`.

3. Mayes, Benjamin Walker. *Tetrahedral Symmetry, Radial Spectra and Finite Collective Mechanisms*. Paper III, consolidated review draft 1.1.1, 4 October 2026. M5 release, `source/paper-3.md`.

4. Mayes, Benjamin Walker. *Finite Prepared Quantum Controllers, Capture Windows and Correlation-Preserving Renewal*. Paper IV, development draft 0.1, 4 October 2026. M5 release, `source/paper-4.md`.

5. *Governance Amendment — CRL-2 Subdivision and Anti-Erosion Rules*, supplied `UD_GOVERNANCE_CRL2_SUBDIVISION_v1.0.md`. Development and registry lanes remain

distinct.

6. Mayes UD research record. *Native pre-event projection obstruction*, 12 October 2026, `captures/2026-10-12T014919Z-codex-native-source-97c0393f/THEOREM.md`.

7. Mayes UD research record. *Direct-Theta event to typed closure and hard-mask pushforward*, 10 October 2026, `captures/2026-10-10T170752Z-codex-theta-closure-245626d6/REPORT.md`.

8. Mayes UD research record. *Bounded subdivision two-event gate* and *Vertex interface and cross-event generator gate*, 13 October 2026, repository captures identified by the companion source register; reports are included in the replay sources.

9. *UD first closure package*, 14 October 2026. `UD_FIRST_CLOSURE_PACKAGE_v0.1.md`, accompanying source hashes, exact incidence certificates and replay.

10. *UD core methodological governance: representation and approximation*, version 1.0, 14 October 2026. User-authorized methodological addition; no theorem-confidence or physical-promotion receipt. `UD_Representation_And_Approximation_Governance_v1.0.md`.

11. Horak, Danijela, and Jürgen Jost. *Spectra of combinatorial Laplace operators on simplicial complexes*. Advances in Mathematics 244 (2013), 303-336. DOI: 10.1016/j.aim.2013.05.007. arXiv:1105.2712.

12. Knill, Oliver. *The Dirac operator of a graph*. 2013. arXiv:1306.2166.

13. Singer, Amit. *Angular Synchronization by Eigenvectors and Semidefinite Programming*. Applied and Computational Harmonic Analysis 30 (2011), 20-36. DOI: 10.1016/j.acha.2010.02.001. arXiv:0905.3174.

14. Christandl, Matthias, Nilanjana Datta, Artur Ekert, and Andrew J. Landahl. *Perfect State Transfer in Quantum Spin Networks*. Physical Review Letters 92 (2004), 187902. DOI: 10.1103/PhysRevLett.92.187902. arXiv:quant-ph/0309131.

15. Reeb, David, and Michael M. Wolf. *An improved Landauer Principle with finite-size corrections*. New Journal of Physics 16 (2014), 103011. DOI: 10.1088/1367-2630/16/10/103011. arXiv:1306.4352.

16. Jacobson, Ted. *Thermodynamics of Spacetime: The Einstein Equation of State*. Physical Review Letters 75 (1995), 1260-1263. DOI: 10.1103/PhysRevLett.75.1260. arXiv:gr-qc/9504004.

17. Serre, Jean-Pierre. *Linear Representations of Finite Groups*. Graduate Texts in Mathematics 42, Springer, 1977. DOI: 10.1007/978-1-4684-9458-7.

18. Knill, Oliver. *The McKean-Singer Formula in Graph Theory*. 2013, §9.3 and complete-graph example. arXiv:1301.1408.

19. Wee, Junjie, Ginestra Bianconi, and Kelin Xia. *Persistent Dirac for molecular representation*. Scientific Reports 13, 11183 (2023). DOI: 10.1038/s41598-023-37853-z.

20. Åström, Karl Johan, and Richard M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Author draft v2.4a, 16 September 2006, §7.1, Theorem 7.1. Inspected draft. Background reference for the observability rank criterion; draft numbering is version-specific.

21. Gouasmi, Ayoub, Eric Parish, and Karthik Duraisamy. *A priori estimation of memory effects in coarse-grained nonlinear systems using the Mori-Zwanzig formalism*. arXiv:1611.06277v2, 2017. DOI: 10.1098/rspa.2017.0385. Background reference for projected memory equations.

22. LAPACK authors. *LAPACK Users' Guide*, "Linear Least Squares Problems" and "How to Measure Errors." Official documentation: least squares, error measures. Background for standard least-squares and norm-based conditioning; the specialized bounds here are proved explicitly.

23. Supplied partner-LLM working notes, *Calculating the Square Root of 4/3*, DOCX upload ingested 14 October 2026. Unreviewed source data; no public-paper or model-identity claim. Original SHA-256, paragraph anchors, dispositions and independent replay are documented in the companion partner-work audit and version-0.9 package. This reference identifies provenance, not authority for the corrected results.

24. Additional supplied partner working notes, `1, 2, and 3(5).docx`, `1, 2, and 3(7).docx` and `1, 2, and 3(8).docx`, ingested 14 October 2026. Exact hashes, paragraph anchors, source/dependency dispositions and independent checks are in the companion additional-research audit and version-0.10 package. Unreviewed source data; this reference records provenance rather than authority for their conclusions.

25. *UD Fine History and Winding Receipts*, version 0.1, 14 October 2026. Independently authored development analysis with a declared graph, normalized kernel family, scalar no-loop proof, exact constraint certificate and finite receipt-size bounds. Companion report and replay in the version-0.11 package. Constructor checked; no registry or physical promotion and no independent review.

26. *UD Additional Research Audit: Affine Control Closure, Phase Extensions and Conditional Transport*, version 0.3, 14 October 2026. Companion constructor audit and independent exact replay for supplied partner working notes `1, 2, and 3(9).docx`, `(10).docx` and `(11).docx`. Original byte hashes, paragraph anchors and equations are preserved in the version-0.12 package. Unreviewed inputs; no independent review or physical promotion.

27. *Carrier-driven Admission: A Passive Hysteresis Candidate*, version 0.1, 15 October 2026 UTC. Companion development report, explicit response contract, written bounds and `verify_admission.py` in the version-0.13 package. Constructor checks only; no independent review, registry conclusion or physical promotion.

28. Li, Zhanjie, Yajing Ma, Dong Yue and Jun Zhao. "Guaranteed Dwell-Times of Hysteresis Switching for Switched Nonlinear Systems via Switching Characteristic Indices." *SIAM Journal on Control and Optimization*. DOI 10.1137/22M1471699. Background attribution for established hysteresis/dwell methods; D.15 derives its own bounds directly. Publisher abstract inspected; no uninspected theorem is imported.

29. *Sampled Event Acquisition and Uncertainty Receipts*, version 0.1, 15 October 2026 UTC. Companion development report, written conditional detection/order bounds, exact finite trace and `verify_sampled_acquisition.py` in the version-0.14 package. Constructor-checked passive observer; no apparatus execution, independent review, registry conclusion or physical promotion.

30. *Additional Research Audit: Receipt Covariance and Twisted Phase Labels*, version 0.4, 15 October 2026 UTC. Companion audit of supplied working notes `1, 2, and 3(12).docx` through `(15).docx`, with original hashes, duplicate-body evidence, claim dispositions, exact covariance certificates and independently authored replay. Primary topology references are specified in the companion report. Development; no independent review, native physical identification or promotion.